RK METHOD FOR SECOND ORDER DIFFERENTIAL EQUATION

To find the soln. of $y'' = f(x, y, y')$ with the given initial cdtn. $y(x_0) = y_0$, $y'(x_0) = y_0'$.

Let $y' = z \Rightarrow \dfrac{dy}{dx} = z = f_1(x, y, z)$

$\Rightarrow y'' = z' \Rightarrow \dfrac{dz}{dx} = y'' = f_2(x, y, z)$ $\Bigg\}$ are simultaneous eqns

$k_1 = h f_1(x_0, y_0, z_0)$ $\qquad$ $l_1 = h f_2(x_0, y_0, z_0)$

$\quad = h z_0$ $\qquad\qquad\qquad$ $= h y_0''$

$k_2 = h f_1\left[x_0 + \dfrac{h}{2},\ y_0 + \dfrac{k_1}{2},\ z_0 + \dfrac{l_1}{2}\right]$ $\qquad$ $l_2 = h f_2\left[x_0 + \dfrac{h}{2},\ y_0 + \dfrac{k_1}{2},\ z_0 + \dfrac{l_1}{2}\right]$

$k_3 = h f_1\left[x_0 + \dfrac{h}{2},\ y_0 + \dfrac{k_2}{2},\ z_0 + \dfrac{l_2}{2}\right]$ $\qquad$ $l_3 = h f_2\left[x_0 + \dfrac{h}{2},\ y_0 + \dfrac{k_2}{2},\ z_0 + \dfrac{l_2}{2}\right]$

$k_4 = h f_1\left[x_0 + h,\ y_0 + k_3,\ z_0 + l_3\right]$ $\qquad$ $l_4 = h f_2\left[x_0 + h,\ y_0 + k_3,\ z_0 + l_3\right]$

$\Delta y = \dfrac{1}{6}\left[k_1 + 2k_2 + 2k_3 + k_4\right]$ $\qquad$ $\Delta z = \dfrac{1}{6}\left[l_1 + 2l_2 + 2l_3 + l_4\right]$

$y_1 = y_0 + \Delta y$ $\qquad\qquad\qquad$ $z_1 = z_0 + \Delta z$

use RK method to determine the approximate value of y at $x = 0.1$ if y satisfies the DE $\dfrac{d^2y}{dx^2} - x^2\dfrac{dy}{dx} - 2xy = 1$. with $y(0) = 1$ and $y'(0) = 0$.

Soln: $y'' = 1 + 2xy + x^2 y'$

with $y(0) = 1$ and $y'(0) = 0$; $h = 0.1$; $x_0 = 0$, $y_0 = 1$; $y_0' = 0$

Let $y' = z \Rightarrow \dfrac{dy}{dx} = z$

$y'' = z' \Rightarrow \dfrac{dz}{dx} = y''$

$\Rightarrow y'' = 1 + 2xy + x^2 z$

with $x_0 = 0$, $y_0 = 1$, $z_0 = 0$

$k_1 = h f_1(x_0, y_0, z_0)$

$\quad = (0.1) z_0$

$\quad = (0.1) 0$

$\quad = 0$

$l_1 = h f_2(x_0, y_0, z_0)$

$\quad = (0.1)[1 + 2x_0 y_0 + x_0^2 z_0]$

$\quad = (0.1)[1 + 2(0) + 0]$

$\quad = 0.1$

$k_2 = hq_1\left[x_0 + \dfrac{h}{2}, y_0 + \dfrac{h_1}{2}, z_0 + \dfrac{l_1}{2}\right]$ $l_2 = hq_2\left[x_0 + \dfrac{h}{2}, y_0 + \dfrac{k_1}{2}, z_0 + \dfrac{l_1}{2}\right]$

$= (0.1)q_1\left[0 + \dfrac{0.1}{2}, 1 + \dfrac{0}{2}, 0 + \dfrac{0.1}{2}\right]$ $l_2 = (0.1)f_2(0.05, 1, 0.05)$

$= (0.1)q_1(0.05, 1, 0.05)$ $= (0.1)\left[1 + 2(0.05)(1) + (0.05)^2\right]$

$= 0.005$ $= 0.1100$

$k_3 = 0.0055$ $l_3 = 0.1100$

$k_4 = 0.0110$ $l_4 = 0.1202$

$\Delta y = 0.0053$ $\Delta z = 0.1100$

$y_1 = y_0 + \Delta y = 1.0053$ $z_1 = z_0 + \Delta z = 0.1100$

HW:

① Consider the second order initial value problem

$y'' - 2y' + 2y = e^{2t}\sin t$ with $y(0) = -0.4$ and $y'(0) = -0.6$

using fourth R.K. method, find $y(0.2)$

<u>soln:</u> $y(0.2) = -0.5159$

② Given $y'' + xy' + y = 0$, $y(0) = 1$, $y'(0) = 0$, find the value

of $y(0.1)$ by using RK method of fourth order.

<u>soln:</u> $y(0.1) = 0.9950$