Subject Code & Name**: 23AST205-Aerospace Structures**

TOPIC: 5. **Neutral axis method Bending equation**

POSITION OF NEUTRAL AXIS

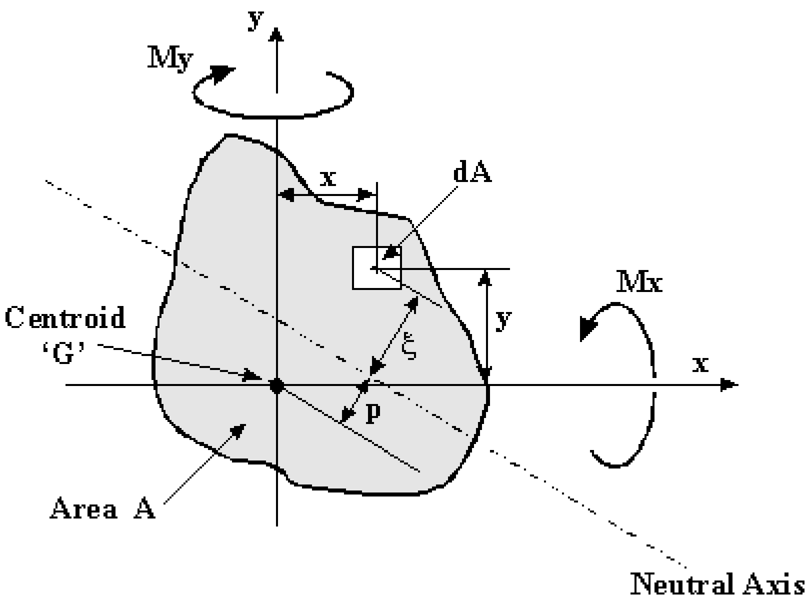


Figure : Determination of Neutral Axis location.

$$\sigma_z = \left(\frac{M_y I_{xx} - M_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2} \right) x + \left(\frac{M_x I_{yy} - M_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2} \right) y \qquad (3.8)$$

By defining the terms ***Effective Bending Moment***, as:

$$\bar{M}_x = \frac{M_x - M_y I_{xy} / I_{yy}}{1 - I_{xy}^2 / I_{xx} I_{yy}} \qquad (3.9)$$

and

$$\bar{M}_y = \frac{M_y - M_x I_{xy} / I_{xx}}{1 - I_{xy}^2 / I_{xx} I_{yy}} \qquad (3.10)$$

Equation 3.8 can be re-written as follows:

$$\sigma_z = \frac{\bar{M}_x}{I_{xx}} y + \frac{\bar{M}_y}{I_{yy}} x \quad \textbf{(3.11)}$$

<u>Note</u> that if the beam is symmetrical about the x -axis or y-axis then:

The location of the Neutral Axis was defined by Equation **(3.3)**.

$$\bar{x} = \frac{\sum \bar{x}_i * A_i}{\sum A_i} \quad , \quad \bar{y} = \frac{\sum \bar{y}_i * A_i}{\sum A_i} \quad \textbf{(3.3)}$$

For a beam with a symmetrical cross section, the centroid is the point defined by equation 3.3 and the Neutral Axis is parallel to the x and y axis. For a non-symmetrical beam cross section however, the Neutral Axis passes at some angle α with respect to the x-axis. What needs to be done define the angle of the neutral axis.

At the N.A. the normal bending stresses are equal to $\sigma_z = 0$, giving that:

$$0 = \frac{\bar{M}_x}{I_{xx}} y_{N.A} + \frac{\bar{M}_y}{I_{yy}} x_{N.A.}$$

where:

$x_{N.A.}$ and $y_{N.A.}$ are coordinates of points along the neutral axis, giving:

$$\frac{y_{N.A.}}{x_{N.A}} = -\frac{\bar{M}_y}{\bar{M}_x} \frac{I_{xx}}{I_{yy}}$$

By taking the inverse tan of the angle 'α', the angle of the Neutral Axis with respect to the x-axis can be found, given by equation 3.12.

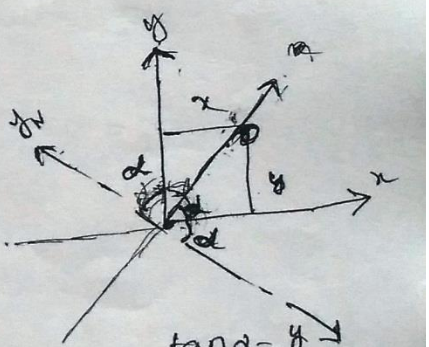
$$\tan(\alpha) = -\frac{\bar{M}_y}{\bar{M}_x} \frac{I_{xx}}{I_{yy}} \quad \textbf{(3.12)}$$

Position of Neutral axis

The Neutral axis always passes through the centroid of area of a beam c/s. but its inclination α to the x axis depends on the form of applied loading and geometric properties of beam c/s.

At all the points on the NA $\sigma_z = 0$

$$\frac{\bar{M}_x}{I_{xx}} y + \frac{\bar{M}_y}{I_{yy}} x = 0$$



$$\frac{y}{x} = -\frac{\bar{M}_y}{\bar{M}_x} \frac{I_{xx}}{I_{yy}}$$

$\tan\alpha = \frac{y}{x}$

$$\tan\alpha = -\frac{\bar{M}_y}{\bar{M}_x} \cdot \frac{I_{xx}}{I_{yy}}$$

$$y_N = y\cos\alpha + x\cos(90+\alpha)$$

$$= y\cos\alpha - x\sin\alpha$$

$$\sigma_N = \frac{M_N}{I_{NN}} y_N$$

$$M_N = M_x \cos\alpha - M_y \sin\alpha$$

$$y_N = y\cos\alpha - x\sin\alpha$$

$$I_{NN} = I_{xx}\cos^2\alpha + I_{yy}\sin^2\alpha - 2I_{xy}\sin\alpha\cos\alpha.$$

Y/LF

Orientation. of PA with NA

$$\tan\theta = \frac{I_{x'x'}}{I_{y'y'}}\tan\phi$$

$$\sigma_p = 0 \qquad \frac{M_x'}{I'_{xx}}\,y + \frac{M_y'}{I'_{yy}}\,x = 0$$

$$\frac{y}{x} = -\frac{M'_y\, I'_{xx}}{M'_x\, I'_{yy}}$$

$$\tan\alpha = \frac{I'_{xx}}{I'_{yy}}\tan\phi.$$