



DEPARTMENT OF MATHEMATICS

UNIT - III

COMPLEX DIFFERENTIATION

INTRODUCTION:

If x and y are real numbers then $z = x + iy$ is called a complex number where x is called real part of z , y is called the imaginary part of z and the value of i is $\sqrt{-1}$. The complex number $x - iy$ is called as the complex conjugate of z & it is denoted by \bar{z} .
i.e., $\bar{z} = x - iy$.

NOTE:

1. $|z| = \sqrt{x^2 + y^2}$
2. $|z^2| = z\bar{z}$
3. $z\bar{z} = x^2 + y^2 = r^2$
4. $|\bar{z}| = |z|$
5. Real part of $z = \frac{z + \bar{z}}{2}$
6. Imaginary part of $z = \frac{z - \bar{z}}{2i}$
7. $z = re^{i\theta}$ is called polar form of z .
8. Amplitude of $z = \theta = \tan^{-1}(y/x)$

FUNCTIONS OF COMPLEX VARIABLE:

$w = f(z) = u(x, y) + iv(x, y)$ where $u(x, y)$ and $v(x, y)$ are real variables.



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Single Valued function:

If for each value of z in R there will be only one value of w , then w is called a single valued function of z .

Ex: $w = z^2$, $w = 1/z$.

$z : 1 \quad 2 \quad -2 \quad 3$

$w : 1 \quad 4 \quad 4 \quad 9$

$z : 1 \quad 2 \quad -2 \quad 3$

$w : 1 \quad 1/2 \quad -1/2 \quad 1/3$

Multiple - valued function:

If there is more than one value of w corresponding to a given value of z , then w is called a multiple-valued function.

Ex: $w = z^{1/2}$

$z : 4 \quad 9 \quad 1$

$w : -2, 2 \quad -3, 3 \quad 1, -1$

Analytic function:

A function $f(z)$ is said to be analytic at a point $z = a$ in a region R if

- (i) $f(z)$ is differentiable at $z = a$.
- (ii) $f(z)$ is differentiable at all points for some neighbourhood of $z = a$.

(or)

A function is said to be analytic at a point if its derivative exists not only at that point but also in some neighbourhood of that point.