SNS COLLEGE OF TECHNOLOGY

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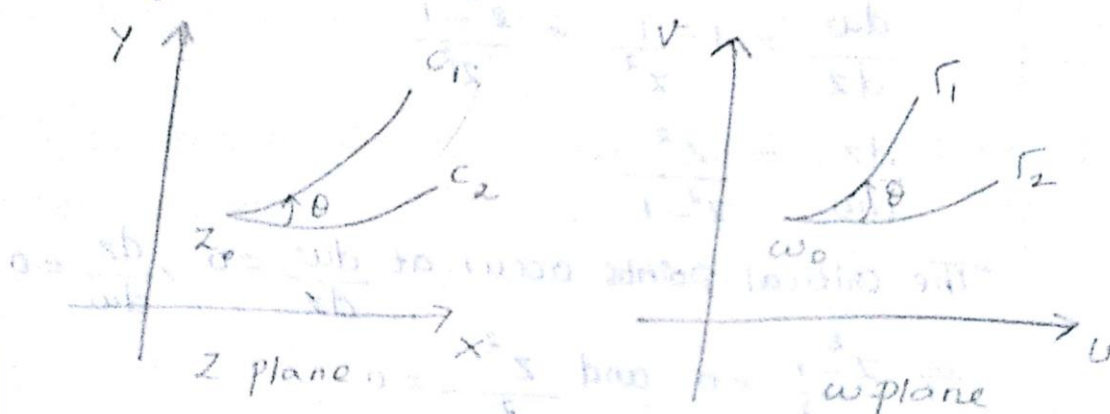*COIMBATORE-641 035, TAMIL NADU*

DEPARTMENT OF MATHEMATICS

Conformal mapping :

Defn:

A mapping $w = f(z)$ is said to be Conformal at $z = z_0$ if it preserves the angle between any two curves through z_0 in z plane both in magnitude and direction.
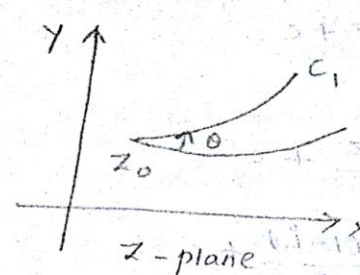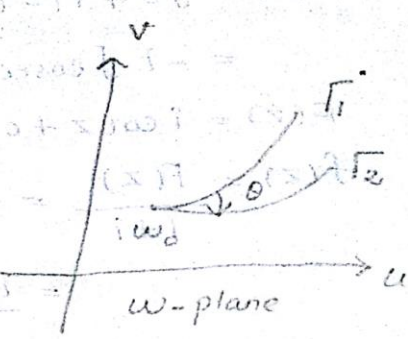

Isogonal mapping :

A mapping $w = f(z)$ is said to be isogonal at $z = z_0$ if it preserves the angle between any two curves through z_0 in z plane only in magnitude

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but not in direction.

Y

c_1

c_2

z_0 θ

z - plane

V

Γ_1

$\sqrt{2}$

iw_0 θ $\sqrt{2}$

w - plane

Remarks:

1. If $f(z)$ is analytic and $f'(z) \neq 0$ at each Point then the mapping $w = f(z)$ is conformal.

2. The Points at which $w = f(z)$ is not conformal i.e., $f'(z) = 0$ are called Critical points.

3. The Critical Points of $w = f(z)$ will occur at $\frac{dz}{dw} = 0$ and $\frac{dw}{dz} = 0$.

① Find the critical points of the transformation $w = z + 1/z$.

Soln:

$$\frac{dw}{dz} = 1 - \frac{1}{z^2} = \frac{z^2 - 1}{z^2}$$

$$\frac{dz}{dw} = \frac{z^2}{z^2 - 1}$$

The critical points occur at $\frac{dw}{dz} = 0$, $\frac{dz}{dw} = 0$.

$$\Rightarrow \frac{z^2 - 1}{z^2} = 0 \quad \text{and} \quad \frac{z^2}{z^2 - 1} = 0$$

$$\Rightarrow z^2 - 1 = 0 \quad \text{and} \quad z^2 = 0$$

$$\Rightarrow z = \pm 1 \quad \text{and} \quad z = 0$$

∴ $z = 0, 1, -1$ are the critical points.

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② Find the Critical points of $w^2 = (z-\alpha)(z-\beta)$.

Soln:

$$w^2 = (z-\alpha)(z-\beta)$$

$$2w\frac{dw}{dz} = (z-\alpha) + (z-\beta)$$

$$\frac{dw}{dz} = \frac{(z-\alpha) + (z-\beta)}{2w}$$

$$\frac{dz}{dw} = \frac{2w}{(z-\alpha) + (z-\beta)}$$

$$\frac{dw}{dz} = \frac{(z-\alpha) + (z-\beta)}{2w} \quad \& \quad \frac{dz}{dw} = \frac{2w}{(z-\alpha) + (z-\beta)}$$

∴ The Critical points occur at $\frac{dw}{dz} = 0$ and $\frac{dz}{dw} = 0$.

$$\Rightarrow \frac{z-\alpha + z-\beta}{2w} = 0 \quad \text{and} \quad \frac{2w}{(z-\alpha) + (z-\beta)} = 0$$

$$\Rightarrow 2z = \alpha + \beta \qquad\qquad w = 0$$

$$\Rightarrow z = \frac{\alpha+\beta}{2} \qquad\qquad \Rightarrow w^2 = 0$$

$$\Rightarrow (z-\alpha)(z-\beta) = 0$$

$$\Rightarrow z = \alpha, \beta$$

∴ $z = \alpha, \beta, \dfrac{\alpha+\beta}{2}$ are the Critical points.

③ Find the points such that $w = f(z) = \sin z$ is not conformal.

Soln: Let $w = \sin z$

$$\frac{dw}{dz} = \cos z \quad ; \quad \frac{dz}{dw} = \frac{1}{\cos z}$$

The Critical points occur at $\frac{dw}{dz} = 0$ and $\frac{dz}{dw} = 0$.

$$\Rightarrow \cos z = 0 \quad \text{and} \quad 1/\cos z = 0$$

$$\Rightarrow z = \cos^{-1}(0) \qquad\qquad 1 = 0, \text{ impossible}$$

$$z = \pm (2n-1)\frac{\pi}{2}, \quad n = 0, 1, 2, \cdots$$