



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

COIMBATORE-641 035, TAMIL NADU



DEPARTMENT OF MATHEMATICS

Bilinear transformation

$$\frac{(w-w_1)(w_2-w_3)}{(w-w_3)(w_2-w_1)} = \frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}$$

$$\frac{(w-w_3)(w_2-w_1)}{(w-w_1)(w_2-w_3)} = \frac{(z-z_3)(z_2-z_1)}{(z-z_1)(z_2-z_3)}$$

Type I: One of the value of z -plane or w -plane is ∞ .

- (1) Find the bilinear transformation that maps the points $z = 0, 1, \infty$ into $w = i, 1, -i$ respl-

Soln:

$$z_1 = 0, z_2 = 1, z_3 = \infty, w_1 = i, w_2 = 1, w_3 = -i$$

$$w_1 = i, w_2 = 1, w_3 = -i$$

$$\frac{(w-i)(1+i)}{(w+i)(1-i)} = \frac{(z-0)(z-\infty)}{(z-\infty)(z-1)} = \frac{z}{1-z}$$

$$\frac{(w-i)(1+i)}{(w+i)(1-i)} = \frac{z}{1-z}$$

$$\frac{(w-i)(1+i)}{(w+i)(1-i)} \times \frac{1+i}{1-i} = \frac{z}{1-z} \times \frac{1+i}{1-i}$$

$$\frac{(w-i)(1+2i-1)}{(w+i)(1-1)} = \frac{z(1+i)}{1-z}$$

$$\frac{(w-i)2i}{(w+i)0} = \frac{z(1+i)}{1-z}$$

$$\frac{(w-i)2i}{2(w+i)} = \frac{z(1+i)}{1-z}$$

$$\frac{(w-i)i}{w+i} = \frac{z(1+i)}{1-z}$$

$$(w-i)i = z(w+i)$$

$$wi + 1 = zw + zi$$

$$wi - zw = zi - 1$$

$$w(i-z) = zi - 1$$

$$w = \frac{iz - 1}{i - z}$$



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DEPARTMENT OF MATHEMATICS

Type 2: No Value is ∞

- ① Find the bilinear transform mapping the points $z = 1, i, -1$ into the points $w = 2, i, -2$ resp.

Soln:

$$\frac{(w-2)(i+2)}{(w+2)(i-2)} = \frac{(z-1)(i+1)}{(z+1)(i-1)}$$

$$\frac{(w-2)}{(w+2)} = \frac{(z-1)(i+1)(i-2)}{(z+1)(i-1)(i+2)}$$

$$= \frac{(z-1)(-3-i)}{(z+1)(i-3)}$$

$$\frac{w-2}{w+2} = + \frac{(z-1)(3+i)}{(z+1)(3-i)}$$

By Componendo - Dividendo rule,

$$\left[\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d} \right]$$

$$\frac{w-2+w+2}{w-2-w-2} = \frac{(z-1)(3+i) + (z+1)(3-i)}{(z-1)(3+i) - (z+1)(3-i)}$$

$$\frac{2w}{-4} = \frac{6z-2i}{2(iz-3)}$$

$$\boxed{w = \frac{-6z + 2i}{iz - 3}}$$

- ② $z = 1, i, -1$ onto $w = i, 0, -i$

Soln:

$$\boxed{w = \frac{1+iz}{1-iz}}$$