

**DEPARTMENT OF MATHEMATICS**

$$\begin{aligned}
 &= \int_0^{\infty} \left[\int_s^{\infty} e^{-st} f(t) ds \right] dt \quad \text{--- 2nd pol} \\
 &= \int_0^{\infty} f(t) \left[\int_s^{\infty} e^{-st} ds \right] dt \quad \text{--- 2nd pol} \\
 &= \int_0^{\infty} f(t) \left[\frac{e^{-st}}{-s} \right]_s^{\infty} dt \quad \text{--- 1st pol} \\
 &= \int_0^{\infty} f(t) \left[0 - \frac{e^{-st}}{-s} \right] dt \quad \text{--- 1st pol} \\
 &= \int_0^{\infty} e^{-st} \frac{f(t)}{s} dt \quad \text{--- 2nd pol} \\
 &= \mathcal{L} \left[\frac{f(t)}{t} \right] \\
 \therefore \quad \mathcal{L} \left[\frac{f(t)}{t} \right] &= \int_s^{\infty} F(s) ds.
 \end{aligned}$$

Problems :

① Find $\mathcal{L} \left(\frac{1 - \cos t}{t} \right)$

Soln:

$$\begin{aligned}
 \mathcal{L} \left(\frac{1 - \cos t}{t} \right) &= \int_s^{\infty} \mathcal{L} [1 - \cos t] ds \\
 &= \int_s^{\infty} \{ \mathcal{L}(1) - \mathcal{L}(\cos t) \} ds \\
 &= \int_s^{\infty} \left[\frac{1}{s} - \frac{s}{s^2 + 1} \right] ds \\
 &= \left[\log s - \frac{1}{2} \log(s^2 + 1) \right]_s^{\infty}
 \end{aligned}$$



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DEPARTMENT OF MATHEMATICS

$$= \left[\log s - \log (s^2+1)^{1/2} \right]_s^\infty$$

$$= \left[\log \frac{s}{\sqrt{s^2+1}} \right]_s^\infty$$

$$= \left[\log \frac{1}{\sqrt{1+\frac{1}{s^2}}} \right]_s^\infty$$

$$= \log 1 - \log \left[\frac{1}{\sqrt{1+\frac{1}{s^2}}} \right]$$

$$= 0 - \log \frac{s}{\sqrt{s^2+1}}$$

$$= \log \left[\frac{s}{\sqrt{s^2+1}} \right]^{-1}$$

$$= \log \left[\frac{\sqrt{s^2+1}}{s} \right]$$

(2) Find $L \left(\frac{e^{-3t} - e^{-4t}}{t} \right)$

Soln:

$$L(e^{-3t} - e^{-4t}) = \frac{1}{s+3} - \frac{1}{s+4}$$

$$L \left[\frac{e^{-3t} - e^{-4t}}{t} \right] = \int_s^\infty \left(\frac{1}{s+3} - \frac{1}{s+4} \right) ds$$

$$= \int_s^\infty \left(\frac{1}{s+3} - \frac{1}{s+4} \right) ds$$

$$= \left[\log(s+3) - \log(s+4) \right]_s^\infty$$

$$= \left[\log \left(\frac{s+3}{s+4} \right) \right]_s^\infty = \log \left(\frac{s+4}{s+3} \right)$$



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③ Find $\mathcal{L} \left[\frac{1 - \cos at}{t} \right]$

Soln:

$$\begin{aligned} \mathcal{L} \left[\frac{1 - \cos at}{t} \right] &= \int_s^\infty \mathcal{L} (1 - \cos at) ds \\ &= \int_s^\infty \left[\frac{1}{s} - \frac{s}{s^2 + a^2} \right] ds \\ &= \left[\log s - \frac{1}{2} \log (s^2 + a^2) \right]_s^\infty \\ &= \left[\log \frac{s}{\sqrt{s^2 + a^2}} \right]_s^\infty \\ &= 0 - \log \left(\frac{s}{\sqrt{s^2 + a^2}} \right) \\ &= \log \left(\frac{\sqrt{s^2 + a^2}}{s} \right) \end{aligned}$$

④ Find $\mathcal{L} \left[\frac{\cos at - \cos bt}{t} \right]$

Soln:

$$\begin{aligned} \mathcal{L} \left[\frac{\cos at - \cos bt}{t} \right] &= \int_s^\infty \mathcal{L} [\cos at - \cos bt] ds \\ &= \int_s^\infty \left[\frac{a}{s^2 + a^2} - \frac{b}{s^2 + b^2} \right] ds \\ &= \frac{1}{2} \left[\log (s^2 + a^2) - \log (s^2 + b^2) \right]_s^\infty \\ &= \frac{1}{2} \left[\log \frac{s^2 + a^2}{s^2 + b^2} \right]_s^\infty \\ &= \frac{1}{2} \left[0 - \log \frac{s^2 + a^2}{s^2 + b^2} \right] \\ &= \frac{1}{2} \log \frac{s^2 + b^2}{s^2 + a^2} \end{aligned}$$



DEPARTMENT OF MATHEMATICS

⑤ Find the Laplace transform of $e^{-t} \int_0^t t \cos t \, dt$

Soln:

$$\mathcal{L} \left[e^{-t} \int_0^t t \cos t \, dt \right] = \left[\mathcal{L} \left(\int_0^t t \cos t \, dt \right) \right]_{s \rightarrow s+1}$$

$$\left(\because \mathcal{L} \int_0^t f(t) \, dt = \frac{1}{s} \mathcal{L}[f(t)] \right)$$

$$= \left[\frac{1}{s} \mathcal{L}(t \cos t) \right]_{s \rightarrow s+1}$$

$$= \left[\frac{1}{s} \left(-\frac{d}{ds} \mathcal{L}(\cos t) \right) \right]_{s \rightarrow s+1}$$

$$= \left[-\frac{1}{s} \frac{d}{ds} \left(\frac{s}{s^2+1} \right) \right]_{s \rightarrow s+1}$$

$$= \left[-\frac{1}{s} \left(\frac{s^2+1-2s^2}{(s^2+1)^2} \right) \right]_{s \rightarrow s+1}$$

$$= \left[-\frac{1}{s} \left(\frac{1-s^2}{(s^2+1)^2} \right) \right]_{s \rightarrow s+1}$$

$$= \left[\frac{s^2-1}{s(s^2+1)^2} \right]_{s \rightarrow s+1}$$

$$= \frac{(s+1)^2-1}{(s+1)((s+1)^2+1)^2}$$

$$= \frac{s^2+2s}{(s+1)(s^2+2s+2)^2}$$

⑥ Evaluate using Laplace transform

$$\int_0^{\infty} t e^{-2t} \sin 3t \, dt$$



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DEPARTMENT OF MATHEMATICS

⑤ Find the Laplace transform of $e^{-t} \int_0^t t \cos t \, dt$

Soln:

$$L \left[e^{-t} \int_0^t t \cos t \, dt \right] = \left[\frac{1}{s} \left(\int_0^t t \cos t \, dt \right) \right]_{s \rightarrow s+1}$$

$$\left(\because \frac{1}{s} \int_0^t f(t) \, dt = \frac{1}{s} L[f(t)] \right)$$

$$= \left[\frac{1}{s} L(t \cos t) \right]_{s \rightarrow s+1}$$

$$= \left[\frac{1}{s} \left(-\frac{d}{ds} L(\cos t) \right) \right]_{s \rightarrow s+1}$$

$$= \left[-\frac{1}{s} \frac{d}{ds} \left(\frac{s}{s^2+1} \right) \right]_{s \rightarrow s+1}$$

$$= \left[-\frac{1}{s} \left(\frac{s^2+1-2s^2}{(s^2+1)^2} \right) \right]_{s \rightarrow s+1}$$

$$= \left[-\frac{1}{s} \left(\frac{1-s^2}{(s^2+1)^2} \right) \right]_{s \rightarrow s+1}$$

$$= \left[\frac{s^2-1}{s(s^2+1)^2} \right]_{s \rightarrow s+1}$$

$$= \frac{(s+1)^2-1}{(s+1)((s+1)^2+1)^2}$$

$$= \frac{s^2+2s}{(s+1)(s^2+2s+2)^2}$$

⑥ Evaluate using Laplace transform

$$\int_0^\infty t e^{-2t} \sin 3t \, dt$$