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DEPARTMENT OF MATHEMATICS

Initial Value theorem:
If the Laplace thans form of
$$f(t)$$
 and $f'(t)$
exists and $L[f(t)] = F(s)$ then
 $Lt = f(t)] = Lt = [S F(s)]$
Proof:
We know that
 $L[f'(t)] = S L[f(t)] - f(o)$
 $= S F(s) - f(o) = S F(s) = L[f'(t)] + f(o)$
 $S F(s) = \int_{0}^{\infty} e^{-St} f'(t) dt + f(o)$
Taking limit as $S \to \infty$ on both sides we get,
 $Lt = SF(s) = Lt = \int_{0}^{\infty} e^{-St} f'(t) dt + f(o)g$





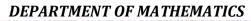
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DEPATMENT OF MATHEMATICS $= \underbrace{lt}_{S \to \infty} \int e^{-St} f'(t) dt + f(0)$ $= \int_{0}^{\infty} \frac{1t}{5 \to \infty} e^{-5t} f'(t) dt + f(0)$ - 0 + f(0) $= \frac{1t}{t \rightarrow 0} - f(t)$ Hence Lt = f(t) = Lt = SF(S) $t \to 0 \qquad S \to \infty$ Final value theorem : If the Laplace transform of f(t) and f'(t) exists and L[f(t)] = F(S) then $\begin{array}{c} \text{Lt} \\ \text{t} \rightarrow \infty \end{array} \begin{bmatrix} f(t) \end{bmatrix} = \begin{array}{c} \text{Lt} \\ \text{s} \rightarrow \infty \end{array} \begin{bmatrix} s & F(s) \end{bmatrix}$ Proof : We know that L [f'(t)] = S L [f(t)] - f(0)= SF(S) = -f(0) both of a laws. \Rightarrow SF(S) = $\bot [f'(t)] + f(0)$ Taking Limit s -> 0 on both sides, we get, $\begin{array}{cccc} 1t & [S F(s)] = 1t & S \int e^{-St} f'(t) dt + f(o) \\ S \rightarrow o & \end{array}$ $= \int_{s \to 0}^{t} \frac{t}{e^{-st}} f'(t) dt + f(0)$ $= \int_{-\infty}^{\infty} f'(t) dt + f(0)$ $= [f(t)]_{0}^{\infty} + f(0)$ $= f(\infty) - f(0) + f(0) = Lt f(t)$ Hence Lt = f(t) = Lt [SF(s)]





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O Vesify the initial and final value theorem for

$$f(t) = 1 + e^{t} (Sint + cost)$$

$$\frac{Soln:}{F(S)} = L \left[1 + e^{t} Sint + e^{t} cost\right]$$

$$= L(1) + L(Sint)_{S \to S+1} + L (cost)_{S \to S+1}$$

$$= \frac{1}{5} + \left(\frac{1}{5^{2}+1}\right)_{S \to S+1} + \left(\frac{5}{5^{2}+1}\right)_{S \to S+1}$$

$$= \frac{1}{5} + \frac{1}{(s+1)^{2}+1} + \frac{5+1}{(s+1)^{2}+1}$$

$$= \frac{1}{5} + \frac{5+2}{5^{2}+25+2}$$
Initial value theorem:

$$\frac{Lt}{t \to 0} = f(t) = \frac{Lt}{t \to 0} \left[1 + e^{t} (Sint + cost)\right] = 1 + 1 = 2$$

$$\frac{Lt}{S \to \infty} S F(s) = \frac{Lt}{S \to \infty} S \left[\frac{1}{5} + \frac{5+2}{5^{2}+25+2}\right]$$

$$= \frac{Lt}{1+1}$$
Hence $\frac{Lt}{t \to 0} = \frac{1}{5} + \frac{1}{5 \to \infty} S F(s) = 2$

$$Tv \tau is Vesified.$$
Final value theorem:

$$\frac{Lt}{t \to \infty} f(t) = \frac{Lt}{t \to \infty} \left[1 + e^{t} (Sint + cost)\right] = 1$$





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DEPARTMENT OF MATHEMATICS

Lt
$$S F(s) = Lt$$
 $S \left[\frac{1}{s} + \frac{s+a}{s^2+as+a} \right]$
 $= \frac{Lt}{s \to o} \left[1 + \frac{s^2+as}{s^2+as+a} \right] = 1$
Hence $Lt = f(t) = Lt$ $S F(s) = 1$, Fort is Varified
Laplace Transform of Some Special functions:
Unit Step function:
The unit step function also called
Heavisides whit function is defined as,
 $U(t-a) = \int_{s} o, t \leq a$
 $This$ is the unit step functions at $t=a$.
The can also be denoted by $H(t-a)$ or $u_a(t)$.
Result:
Laplace Transform of Whit Step function
is $\frac{e}{s}$ i.e., $L [u(t-a)] = \frac{e}{s}$
 $\frac{e}{s}$
 $\frac{1}{s} e^{-st}u(t-a)dt + \int_{a}^{b} e^{-st}u(t-a)dt$
 $= \int_{a}^{b} e^{-st}u(t-a)dt + \int_{a}^{b} e^{-st}u(t-a)dt$