



DEPARTMENT OF MATHEMATICS

Inverse Laplace Transform:

If the Laplace transform of $f(t)$ is $F(s)$ i.e., $L[f(t)] = F(s)$. Then $f(t)$ is called an inverse Laplace transform of $F(s)$ and is written as $f(t) = L^{-1}[F(s)]$ where L^{-1} is called the inverse Laplace transform operator.



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) & amp;

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & amp; B.Tech.IT)

COIMBATORE-641 035, TAMIL NADU



DEPATMENT OF MATHEMATICS

Table of Inverse Laplace Transforms:

$\mathcal{L}[f(t)] = F(s)$	$\mathcal{L}^{-1}[F(s)] = f(t)$
① $\mathcal{L}(1) = \frac{1}{s}$	$\mathcal{L}^{-1}\left(\frac{1}{s}\right) = 1$
② $\mathcal{L}(t) = \frac{1}{s^2}$	$\mathcal{L}^{-1}\left(\frac{1}{s^2}\right) = t$
③ $\mathcal{L}(t^n) = \frac{n!}{s^{n+1}}$	$\mathcal{L}^{-1}\left(\frac{n!}{s^{n+1}}\right) = t^n$
④ $\mathcal{L}(e^{at}) = \frac{1}{s-a}$	$\mathcal{L}^{-1}\left(\frac{1}{s-a}\right) = e^{at}$
⑤ $\mathcal{L}(e^{-at}) = \frac{1}{s+a}$	$\mathcal{L}^{-1}\left(\frac{1}{s+a}\right) = e^{-at}$
⑥ $\mathcal{L}(\sin at) = \frac{a}{s^2+a^2}$	$\mathcal{L}^{-1}\left(\frac{a}{s^2+a^2}\right) = \sin at$
⑦ $\mathcal{L}\left(\frac{\sin at}{a}\right) = \frac{1}{s^2+a^2}$	$\mathcal{L}^{-1}\left(\frac{1}{s^2+a^2}\right) = \frac{\sin at}{a}$
⑧ $\mathcal{L}(\cos at) = \frac{s}{s^2+a^2}$	$\mathcal{L}^{-1}\left(\frac{s}{s^2+a^2}\right) = \cos at$
⑨ $\mathcal{L}(\sinh at) = \frac{a}{s^2-a^2}$	$\mathcal{L}^{-1}\left(\frac{a}{s^2-a^2}\right) = \sinh at$
⑩ $\mathcal{L}(\cosh at) = \frac{s}{s^2-a^2}$	$\mathcal{L}^{-1}\left(\frac{s}{s^2-a^2}\right) = \cosh at$
⑪ $\mathcal{L}[\delta(t)] = 1$	$\mathcal{L}^{-1}(1) = \delta(t)$



DEPARTMENT OF MATHEMATICS

Important Results in Laplace Transforms:

If a and b are constants while $F(s)$ and $G(s)$ are the Laplace transform of $f(t)$ & $g(t)$ respectively.

1. Linearity property:

$$\mathcal{L}^{-1}[a F(s) + b G(s)] = a \mathcal{L}^{-1}[F(s)] + b \mathcal{L}^{-1}[G(s)]$$

2. First Shifting property:

$$(i) \mathcal{L}^{-1}[F(s+a)] = e^{-at} \mathcal{L}^{-1}[F(s)]$$

$$(ii) \mathcal{L}^{-1}[F(s-a)] = e^{at} \mathcal{L}^{-1}[F(s)]$$

3. Second Shifting property:

If $\mathcal{L}^{-1}[F(s)] = f(t)$ then

$$\mathcal{L}^{-1}[e^{-as} F(s)] = g(t) \text{ where}$$

$$g(t) = \begin{cases} f(t-a), & t > a \\ 0, & t \leq a \end{cases}$$

4. Change of Scale property:

$$\mathcal{L}^{-1}[F(ks)] = \frac{1}{k} f\left(\frac{t}{k}\right)$$

5. Multiplication by s :

If $\mathcal{L}^{-1}[F(s)] = f(t)$ and $f(0) = 0$

$$\text{Then } \mathcal{L}^{-1}[s F(s)] = \frac{d}{dt} \mathcal{L}^{-1}[F(s)]$$

Note:

$$\text{If } f(0) \neq 0 \text{ then } \mathcal{L}^{-1}[s F(s)] = \frac{d}{dt} \mathcal{L}^{-1}[F(s)] + f(0) \delta(t).$$



DEPARTMENT OF MATHEMATICS

Problem Identification:

If $L^{-1} \left[\frac{s}{\text{quadratic equation}} \right]$ then use result 5.

If $L^{-1} \left[\frac{s}{\text{Linear equation}} \right]$ then we use the above note.

6. Division by s:

$$L^{-1} \left[\frac{F(s)}{s} \right] = \int_0^t L^{-1} [F(s)] dt$$

7. Inverse Laplace transform of derivatives:

$$\text{If } L^{-1} [F(s)] = f(t) \text{ then } L^{-1} [F'(s)] = -t L^{-1} [F(s)]$$

Problem Identification:

If $L^{-1} \left[\frac{s + \text{any term}}{(\text{quadratic eqn})^2} \right]$ then we use the above result.

8. Note:

$$\text{If } L^{-1} [F(s)] = f(t) \text{ then } L^{-1} [F(s)] = -\frac{1}{t} L^{-1} \left[\frac{d}{ds} F(s) \right]$$

Problem Identification:

If $L^{-1} [\log \text{ function or cot function or tan fn}]$ then we use the above result.