Inverse Laplace Transform:

If the Laplace transform of $f(t)$ is $F(s)$ i.e., $L[f(t)] = F(s)$. Then $f(t)$ is called an inverse Laplace transform of $F(s)$ and is written as $f(t) = L^{-1}[F(s)]$ where L^{-1} is called the inverse Laplace transform operator.

Table of Inverse Laplace Transforms :

$L[f(t)] = F(s)$	$L^{-1}[F(s)] = f(t)$
(1) $L(1) = \frac{1}{s}$	$L^{-1}\left(\frac{1}{s}\right) = 1$
(2) $L(t) = \frac{1}{s^2}$	$L^{-1}\left(\frac{1}{s^2}\right) = t$
(3) $L(t^n) = \frac{n!}{s^{n+1}}$	$L^{-1}\left(\frac{n!}{s^{n+1}}\right) = t^n$
(4) $L(e^{at}) = \frac{1}{s-a}$	$L^{-1}\left(\frac{1}{s-a}\right) = e^{at}$
(5) $L(e^{-at}) = \frac{1}{s+a}$	$L^{-1}\left(\frac{1}{s+a}\right) = e^{-at}$
(6) $L(\sin at) = \frac{a}{s^2+a^2}$	$L^{-1}\left(\frac{a}{s^2+a^2}\right) = \sin at$
(7) $L\left(\frac{\sin at}{a}\right) = \frac{1}{s^2+a^2}$	$L^{-1}\left(\frac{1}{s^2+a^2}\right) = \frac{\sin at}{a}$
(8) $L(\cos at) = \frac{s}{s^2+a^2}$	$L^{-1}\left(\frac{s}{s^2+a^2}\right) = \cos at$
(9) $L(\sinh at) = \frac{a}{s^2-a^2}$	$L^{-1}\left(\frac{a}{s^2-a^2}\right) = \sinh at$
(10) $L(\cosh at) = \frac{s}{s^2-a^2}$	$L^{-1}\left(\frac{s}{s^2-a^2}\right) = \cosh at$
(11) $L[\delta(t)] = 1$	$L^{-1}(1) = \delta(t)$

Important Results in Laplace Transforms:

If a and b are constants while $F(s)$ and $G(s)$ are the Laplace transform of $f(t)$ & $g(t)$ respectively.

1. Linearity property:

$$L^{-1}[a\,F(s) + b\,G(s)] = a\,L^{-1}[F(s)] + b\,L^{-1}[G(s)]$$

2. First Shifting property:

(i) $L^{-1}[F(s+a)] = e^{-at}\,L^{-1}[F(s)]$

(ii) $L^{-1}[F(s-a)] = e^{at}\,L^{-1}[F(s)]$

3. Second Shifting property:

If $L^{-1}[F(s)] = f(t)$ then

$L^{-1}[e^{-as}F(s)] = g(t)$ where

$$g(t) = \begin{cases} f(t-a), & t > a \\ 0, & t < a \end{cases}$$

4. Change of Scale property:

$$L^{-1}[F(ks)] = \frac{1}{k} f\left(\frac{t}{k}\right)$$

5. Multiplication by s:

If $L^{-1}[F(s)] = f(t)$ and $f(0) = 0$

then $L^{-1}[s\,F(s)] = \frac{d}{dt} L^{-1}[F(s)]$

Note:

If $f(0) \neq 0$ then $L^{-1}[s\,F(s)] = \frac{d}{dt} L^{-1}[F(s)] + f(0)\,\delta(t)$.

Problem Identification:

If $L^{-1}\left[\frac{s}{\text{quadratic equation}}\right]$ then use result 5.

If $L^{-1}\left[\frac{s}{\text{Linear equation}}\right]$ then we use the above note.

6. Division by s:

$$L^{-1}\left[\frac{F(s)}{s}\right] = \int_0^t L^{-1}[F(s)]\,dt$$

7. Inverse Laplace transform of derivatives:

If $L^{-1}[F(s)] = f(t)$ then $L^{-1}[F'(s)] = -t\,L^{-1}[F(s)]$

Problem Identification:

If $L^{-1}\left[\frac{s + \text{any term}}{(\text{quadratic eqn})^2}\right]$ then we use the above result.

8. Note:

If $L^{-1}[F(s)] = f(t)$ then $L^{-1}[F(s)] = -\frac{1}{t} L^{-1}\left[\frac{d}{ds}F(s)\right]$

Problem Identification:

If L^{-1} [log function or cot function or tan fn] then we use the above result.