

**DEPARTMENT OF MATHEMATICS**

$$\begin{aligned}
 &= \frac{\sin 3t}{3} \left[\frac{e^{-t}}{10} (-\cos 3t + 3 \sin 3t) - \frac{1}{10} (-1) \right] \\
 &\quad - \frac{\cos 3t}{3} \left[\frac{e^{-t}}{10} (-\sin 3t - 3 \cos 3t) - \frac{1}{10} (-3) \right] \\
 &= \left[\frac{e^{-t}}{30} \cdot 3 (\sin^2 3t + \cos^2 3t) \right] + \left[\frac{1}{30} (\sin 3t + 3 \cos 3t) \right] \\
 &= \frac{e^{-t}}{10} + \frac{1}{30} (\sin 3t + 3 \cos 3t)
 \end{aligned}$$

Applications of Laplace transforms to Differential Equations:

If $\mathcal{L}[f(t)] = F(s)$ then

$$\mathcal{L}[y'(t)] = s \mathcal{L}(y) - y(0)$$

$$\mathcal{L}[y''(t)] = s^2 \mathcal{L}(y) - s y(0) - y'(0)$$

- ① Solve the differential equations using LT
 $y'' + 4y' + 4y = e^{-t}$ given that $y(0) = 0$ and $y'(0) = 0$.

Soln: $y'' + 4y' + 4y = e^{-t}$

Taking LT on both sides,

$$\mathcal{L}(y'' + 4y' + 4y) = \mathcal{L}(e^{-t})$$

$$\mathcal{L}(y'') + 4 \mathcal{L}(y') + 4 \mathcal{L}(y) = \frac{1}{s+1}$$

$$[s^2 \mathcal{L}(y) - s y(0) - y'(0)] + 4[s \mathcal{L}(y) - y(0)] + 4 \mathcal{L}(y) = \frac{1}{s+1}$$



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Given: $y(0) = 0, y'(0) = 0$

$$\Rightarrow [s^2 L(y) - sx_0 - 0] + 4 [s L(y) - 0] + 4 L(y) = \frac{1}{s+1}$$

$$\Rightarrow s^2 L(y) + 4s L(y) + 4 L(y) = \frac{1}{s+1}$$

$$\Rightarrow (s^2 + 4s + 4) L(y) = \frac{1}{s+1}$$

$$\Rightarrow L(y) (s+2)^2 = \frac{1}{s+1}$$

$$L(y) = \frac{1}{(s+1)(s+2)^2}$$

$$y = L^{-1} \left[\frac{1}{(s+1)(s+2)^2} \right]$$

$$\frac{1}{(s+1)(s+2)^2} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{(s+2)^2}$$

$$1 = A(s+2)^2 + B(s+2)(s+1) + C(s+1)$$

Put $s = -2 \Rightarrow C = -1$

Put $s = -1 \Rightarrow A = 1$

Put $s = 0 \Rightarrow B = -1$

$$\frac{1}{(s+1)(s+2)^2} = \frac{1}{s+1} - \frac{1}{s+2} - \frac{1}{(s+2)^2}$$

$$y = L^{-1} \left(\frac{1}{(s+1)(s+2)^2} \right)$$

$$= L^{-1} \left(\frac{1}{s+1} \right) - L^{-1} \left(\frac{1}{s+2} \right) - L^{-1} \left(\frac{1}{(s+2)^2} \right)$$

$$= e^{-t} - e^{-2t} - te^{-2t}$$

**DEPARTMENT OF MATHEMATICS**

(2) Solve using LT $\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 9y = 2e^{-3t}$,
 $y(0) = 1, y'(0) = -2$

Soln:

$$y'' + 6y' + 9y = 2e^{-3t}$$

$$\mathcal{L}(y'' + 6y' + 9y) = 2\mathcal{L}(e^{-3t})$$

$$\mathcal{L}(y'') + 6\mathcal{L}(y') + 9\mathcal{L}(y) = 2\mathcal{L}(e^{-3t})$$

$$\{s^2\mathcal{L}(y) - sy(0) - y'(0)\} + 6\{s\mathcal{L}(y) - y(0)\} + 9\mathcal{L}(y) = \frac{2}{s+3}$$

$$\{s^2\mathcal{L}(y) - s + 2\} + 6\{s\mathcal{L}(y) - 1\} + 9\mathcal{L}(y) = \frac{2}{s+3}$$

$$s^2\mathcal{L}(y) + 6s\mathcal{L}(y) + 9\mathcal{L}(y) - (s+4) = \frac{2}{s+3}$$

$$(s^2 + 6s + 9)\mathcal{L}(y) = \frac{2}{s+3} + s+4$$

$$(s+3)^2\mathcal{L}(y) = \frac{2}{s+3} + s+4$$

$$\mathcal{L}(y) = \frac{2}{(s+3)^3} + \frac{s+4}{(s+3)^2}$$

$$y = \mathcal{L}^{-1}\left[\frac{2}{(s+3)^3} + \frac{s+4}{(s+3)^2}\right]$$

$$= 2\mathcal{L}^{-1}\left(\frac{1}{(s+3)^3}\right) + \mathcal{L}^{-1}\left(\frac{s+4}{(s+3)^2}\right)$$

$$= 2\mathcal{L}^{-1}\left(\frac{1}{(s+3)^3}\right) + \mathcal{L}^{-1}\left[\frac{1}{s+3} + \frac{1}{(s+3)^2}\right]$$

$$= 2e^{-3t}\mathcal{L}^{-1}\left(\frac{1}{s^3}\right) + \mathcal{L}^{-1}\left(\frac{1}{s+3}\right) + \mathcal{L}^{-1}\left(\frac{1}{(s+3)^2}\right)$$

$$= 2e^{-3t} \frac{t^2}{2} + e^{-3t} + te^{-3t}$$