Student t-test:

Test for Single Mean

Null Hypothesis:

$H_0 : \mu = \mu_0$

Alternate Hypothesis

$H_1 : \mu \neq \mu_0$

(or) $\mu > \mu_0$

(or) $\mu < \mu_0$

Test Statistic:

1) SD is not given

$$t = \frac{\bar{x} - \mu}{(s/\sqrt{n})}$$

2) SD is given

$$t = \frac{\bar{x} - \mu}{(SD/\sqrt{n-1})}$$

Here $\bar{x} = \frac{1}{n} \Sigma x_i$

$$s^2 = \frac{1}{n-1} \Sigma_i (x_i - \bar{x})^2$$

μ - population Mean

n - Sample size.

* Degrees of freedom:

$\nu = n - 1$

* If $t_{cal} > t_{tab} \rightarrow H_0$ is Rejected

If $t_{cal} < t_{tab} \rightarrow H_0$ is Accepted.

* Confidence limits:

for μ is $\bar{x} \pm t_{\alpha} \frac{s}{\sqrt{n}}$ $\quad \alpha$ is LOS

SD Given Directly

) A Machinist is making engine parts with axle diameters of 0.700 inch. A random sample of 10 parts shows a mean diameter of 0.742 inch with a SD of 0.040 inch. Compute the statistic you would use to test whether the work is good meeting the specification.

Solution:

Given: Sample Size $n = 10 < 30$ [small Sample]

$\bar{x} = 0.742$ inches | $\mu = 0.700$ inches | $SD = 0.040$

we use student t-test.

Null Hypothesis: $H_0 : \mu = \mu_0 = 0.700$

The product is good.

Alternative Hypothesis: $H_1 : \mu \neq 0.700$ [Two-tailed]

Test Statistic:

$$t_{cal} = \frac{\bar{x} - \mu}{(S.D/\sqrt{n-1})} = \frac{0.742 - 0.700}{\left(\frac{0.040}{\sqrt{10-1}}\right)}$$

$$= 3.15$$

$$\boxed{t_{cal} = 3.15}$$

Degrees of freedom: $\nu = n-1 = 10-1 = 9$

LoS = 5%

t_{tab} at 5% with $\nu = 9$ is $t_{tab} = \boxed{2.26}$

$\therefore t_{cal} > t_{tab}$

$\therefore$ H_0 is Rejected.

$\therefore$ The product is not good and did not ~~does not~~ meet specification

2) The mean weekly sales of soap bars in departmental stores was 146.3 bars per store. After an advertising campaign the mean weekly sales in 22 stores for a typical week increased to 153.7 and showed a SD of 17.2. Was the advertising campaign successful.

Solution:

Given: Sample size $n = 22 < 30$ [small sample)

$\bar{x} = 153.7$ | $\mu = 146.3$ | $SD = 17.2$

We use student t-test

Null Hypothesis: $H_0 : \mu = \mu_0 = 146.3$

The advertising campaign was not successful

Alternative Hypothesis: $H_1 : \mu > 146.3$ [one-tailed]

Test Statistic

$$t_{cal} = \frac{\bar{x} - \mu}{(S.D/\sqrt{n-1})} = \frac{153.7 - 146.3}{(17.2/\sqrt{22-1})}$$

$$\boxed{t_{cal} = 1.97}$$

Degrees of freedom: $\nu = n - 1 = 22 - 1 = 21$

LoS - 5%

t_{tab} at 5% with $\nu = 21$, is $t_{tab} = 1.72$

$t_{cal} > t_{tab}$

$\therefore$ H_0 is rejected.

$\therefore$ The campaign was successful.

) A sample of 26 bulbs gives a mean life of
990 hours with a SD of 20 hours. The
manufacturer claims that the mean life of
bulbs is 1000 hours. Is the sample upto the standard.

Solution:

Given: Sample size $n = 26 < 30$ [small sample]

$\bar{x} = 990$ / $\mu = 1000$ / $SD = 20$

We use student t-test.

Null Hypothesis: $H_0 : \mu = \mu_0$

The sample is good standard.

Alternative Hypothesis: $H_1 : \mu < 1000$ [one-tailed test]

Test statistic: $t_{cal} = \dfrac{\bar{x} - \mu}{\left(SD/\sqrt{n-1}\right)} = \dfrac{990 - 1000}{\left(20/\sqrt{26-1}\right)}$

$$\boxed{t_{cal} = -2.5} \qquad |t_{cal}| = 2.5$$

Degrees of freedom : $\gamma = n - 1 = 26 - 1 = 25$

LoS = 5 %

t_{tab} at 5% with $\gamma = 25$ is $t_{tab} = 1.708$

$t_{cal} > t_{tab}$

$\therefore$ H_0 is rejected.

$\therefore$ The sample is of bad standard.