Test of Significance of Difference of Proportions

$$Z_{cal} = \frac{p_1 - p_2}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where $p = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$ and $q = 1 - p$.

Problem 1:

Random Samples of 400 men and 600 women were asked whether they would like to have a flyover near their residence. 200 men and 325 women were in favour of the proposal. Test the hypothesis that proportions of men and women in favour of the proposal are same, at 5% level.

Solution:

Given: $n_1 = 400$ $\quad n_2 = 600$

proportion of men $p_1 = \frac{200}{400} = 0.5$

proportion of women $p_2 = \frac{325}{600} = 0.541$

Null Hypothesis: $H_0 : p_1 = p_2$

There is no significant difference b/w men and women opinion on flyover proposal.

Alternative Hypothesis: $H_1 : p_1 \neq p_2$ (two tailed).

Test statistic $Z_{cal} = \frac{p_1 - p_2}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$

where $p = \dfrac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$

$= \dfrac{400\ (0.5) + 600\ (0.541)}{400 + 600}$

$\boxed{p = 0.525}$

~~q = 1 - 0.525~~ $q = 1 - p$

$= 1 - 0.525$

$\boxed{q = 0.475}$

$\therefore Z_{cal} = \dfrac{0.5 - 0.541}{\sqrt{(0.525)(0.475)\left(\frac{1}{400} + \frac{1}{600}\right)}}$

$= \dfrac{-0.041}{0.032}$

$= -1.34.$

$|Z_{cal}| = 1.34.$

LoS - 5%, two tailed. $Z_{tab} = 1.96.$

$Z_{cal} < \cancel{1.96}\ Z_{tab}$ ∴ We accept the null hypothesis

∴ There is no difference of opinion b/w men and women concerning the flyover proposal.

2) Before an increase in excise duty on tea, 800 persons out of a sample of 1000 persons were found to be tea drinkers. After an increase in duty, 800 people were tea drinkers in a sample of 1200 people. Using standard error of proportion, state whether there is a significant decrease in the consumption of tea after the increase in excise duty.

Solution:

$n_1 = 1000$ $\qquad p_1 = \frac{800}{1000} = 0.8$

$n_2 = 1200$ $\qquad p_2 = \frac{800}{1200} = 0.667.$

Null Hypothesis:

$H_0 : p_1 = p_2$.

There is no significant difference

Alternative hypothesis $H_1 : p_1 > p_2$ (right tailed test).

Test statistic: $Z_{cal} = \frac{p_1 - p_2}{\sqrt{pq\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$

$= \cancel{0.8 - 0.667}$

$$p = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$$

$$= \frac{1000(0.8) + 1200(0.667)}{1000 + 1200}$$

$\boxed{p = 0.727}$

$q = 1 - 0.727$

$\boxed{q = 0.273}$

$$Z_{cal} = \frac{0.8 - 0.667}{\sqrt{0.727 \times 0.273\left(\frac{1}{1000} + \frac{1}{1200}\right)}}$$

$$= \frac{0.8 - 0.667}{0.019}$$

$$Z_{cal} = 7.$$

LoS 5%, one-tailed, $Z_{tab} = 1.64$.

$Z_{cal} > Z_{tab}$, we reject the null hypothesis

$\therefore$ There is a difference in the consumption of tea before and after the increase in excise duty.

3) In a large city A, 20% of a random sample of 900 school boys had a slight physical defect. In another large city B, 18.5% of a random sample of 1600 school days had the same defect. Is the difference between the proportions significant.

Solution:

Given: $n_1 = 900$ $n_2 = 1600$.

$p_1 = \frac{20}{100} = 0.2$ $p_2 = \frac{18.5}{100} = 0.185$

Null Hypothesis: $H_0: p_1 = p_2$

The difference b/w the proportions are not significant.

Alternative Hypothesis : $H_1 : p_1 \neq p_2$ (two tailed)

Test statistic

$$Z_{cal} = \frac{p_1 - p_2}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$p = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = \frac{900(0.2) + 1600(0.185)}{900 + 1600}$$

$$= \frac{180 + 296}{2500} = \frac{476}{2500}$$

$$p = 0.1904$$

$$q = 1 - p = 1 - 0.1904$$

$$Z_{cal} = \frac{0.2 - 0.185}{\sqrt{(0.1904)(0.8096)\left(\frac{1}{900} + \frac{1}{1600}\right)}}$$

$$= \frac{0.015}{0.016}$$

$$Z_{cal} = 0.9375$$

LOS - 5%, two tailed $Z_{tab} = 1.96$

$$Z_{cal} < Z_{tab}$$

~~∴ we reject~~ ∴ We accept Null hypothesis

∴ The difference b/w the two proportions are not significant.