UNIT 3 SOLUTION OF EQUATIONS
NEWTON RAPHSON METHOD

1) Obtain Newton's Iterative formula for finding $\sqrt{N}$ where N is a +ve real no. Hence evaluate $\sqrt{5}$

Let $x = \sqrt{N} \Rightarrow x^2 = N \Rightarrow x^2 - N = 0.$

$F(x) = x^2 - N$

$F'(x) = 2x$

Now $x_{n+1} = x_n - \frac{F(x_n)}{F'(x_n)}$

$= x_n - \frac{x_n^2 - N}{2x_n} = \frac{2x_n^2 - x_n^2 + N}{2x_n}$

$= \frac{x_n^2 + N}{2x_n}$, which is an iterative formula for $\sqrt{N}$

To find $\sqrt{5}$

$x = \sqrt{5}$

$x^2 - 5 = 0$

$F(x) = x^2 - 5$, $F'(x) = 2x$.

$F(0) = -5$ (-ve)

$F(1) = 1 - 5 = -4$

$F(2) = 4 - 5 = -1$ (-ve)

$F(3) = 9 - 5 = 4$ (+ve)

∴ The root lies between 2 & 3

∵ $|F(2)| < |F(3)|$, let us assume that $x_0 = 2$

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Now, $x_{n+1} = \frac{x_n^2+N}{2x_n} \Rightarrow n=0 \quad x_1 = \frac{x_0^2+5}{2x_0} = \frac{4+5}{2(2)} = \frac{9}{4}$

$$x_1 = 2.25$$

$$x_2 = \frac{x_1^2+N}{2x_1} = \frac{(2.25)^2+5}{2(2.25)}$$

$$= 2.2361$$

$$x_3 = \frac{(2.2361)^2+5}{2(2.2361)} = 2.2361$$

$\therefore$ The value of $\sqrt{5} = 2.2361$.

2) Find the iterative formula for finding the value of $\frac{1}{N}$, where N is a real no. using Newton Raphson method. Hence evaluate $\frac{1}{26}$ correct to 4 decimal places.

Let $x = \frac{1}{N}$ (i.e) $N = \frac{1}{x}$

$F(x) = \frac{1}{x} - N$; $F'(x) = \frac{-1}{x^2}$

Now, $x_{n+1} = x_n - \frac{F(x_n)}{F'(x_n)} = x_n - \frac{\frac{1}{x_n} - N}{-1/x_n^2}$

$$= x_n + x_n(1 - Nx_n) = x_n + x_n - Nx_n^2$$

$x_{n+1} = 2x_n - Nx_n^2$, which is the iterative formula.

To find $\frac{1}{26}$, $N=26$

$F(x) = \frac{1}{x} - 26$; $F'(x) = \frac{-1}{x^2}$

$F(0) = -26$ (-ve)

$F(1) = -25$

$F(2) = -25.5$ (-ve)

Let us take $x_0 = \frac{1}{25} = 0.04$, nearer to given N

Let $x_0 = 0.04$

WKT $x_{n+1} = 2x_n - Nx_n^2$

$x_1 = 2(0.04) - 26(0.04)^2$

$x_1 = 0.0384$

$x_2 = 0.0384$

Since x_1 & x_2 are equal, the value of $\frac{1}{26} = 0.0384$

3) Derive Newtons algorithm for finding the p^{th} root of a number N & find the value of $(24)^{1/3}$

Let $x = N^{1/p}$

$x^p = N$

$\Rightarrow x^p - N = 0$

Let $F(x) = x^p - N$; $F'(x) = px^{p-1}$

WKT, $x_{n+1} = x_n - \frac{F(x_n)}{F'(x_n)} = x_n - \frac{x_n^p - N}{px_n^{p-1}}$

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$$x_{n+1} = \frac{Px_n^P - x_n^P + N}{Px_n^{P-1}} = \frac{(P-1)x_n^P + N}{Px_n^{P-1}}$$

To find $(24)^{1/3}$

Here $N = 24$, $P = 3$.

$F(x) = x^P - N$

$F(x) = x^3 - 24$

$F(0) = -24$

$F(1) = -23$

$F(2) = -16$ (-ve)

$F(3) = 3$ (+ve), the root lies between 2 & 3

Since $|F(2)| > |F(3)|$ let us assume $x_0 = 3$

$$x_{n+1} = \frac{(3-1)x_n^3 + 24}{3x_n^{3-1}} = \frac{2x_n^3 + 24}{3x_n^2}$$

$$x_1 = \frac{2x_0^3 + 24}{3x_0^2} = 2.8888$$

$x_2 = 2.8845$

$x_3 = 2.8844$

$x_4 = 2.8844$

Since $x_3 = x_4$, the required root is 2.8844