



UNIT 3 SOLUTION OF EQUATIONS
NEWTON RAPHSON METHOD

3. Solution of Equations

Newton Method (or) Newton Raphson Method

Newton Raphson method is extensively used for analysis of flow in water distribution networks. It is used to find the roots of non linear equations.

Formula:

$$x_{n+1} = x_n - \frac{F(x_n)}{F'(x_n)}, \text{ provided } F'(x_n) \neq 0.$$

Order = 2.

Newton Raphson condition:

$$|F(x)F''(x)| < |F'(x)|^2.$$

Problems:

1. Find the smallest positive root of the eqn

$$x^3 - 2x + 0.5 = 0.$$

$$\text{Let } F(x) = x^3 - 2x + 0.5$$

$$\text{Now } F'(x) = 3x^2 - 2$$

$$F(0) = 0.5$$

$$F(1) = -0.5$$

$\therefore$ The root lies b/w 0 & 1

$$\text{Since } |F(0)| = |F(1)|$$

Let us assume $x_0 = 0$.



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Newton Raphson Formula:-

$$x_{n+1} = x_n - \frac{F(x_n)}{F'(x_n)}$$

Put $n=0$, $x_1 = x_0 - \frac{F(x_0)}{F'(x_0)} = 0 - \frac{0.5}{-2} = 0.25$

Put $n=1$, $x_2 = x_1 - \frac{F(x_1)}{F'(x_1)} = 0.25 - \frac{F(0.25)}{F'(0.25)}$
 $x_2 = 0.2586$

Put $n=2$, $x_3 = x_2 - \frac{F(x_2)}{F'(x_2)}$
 $= 0.2586.$

Since x_2 & x_3 are equal roots, the smallest positive root is 0.2586 .

d) Compute the real root of $x \log x = 1.2$ correct to 3 decimal places using Newton Raphson method.

Let $f(x) = x \log x - 1.2$

$$F'(x) = x\left(\frac{1}{x}\right) + \log x(1) - 0 = 1 + \log x$$

Now $F(0) = -ve$

$$F(1) = -1.2$$

$$F(2) = -0.5979 \quad \therefore \text{The root lies between}$$

$$F(3) = 0.2314 \quad 2 \text{ \& } 3$$



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Since $|F(2)| > |F(3)|$. Let us assume $x_0 = 3$

Newton - Raphson formula,

$$x_{n+1} = x_n - \frac{F(x_n)}{F'(x_n)}$$

$$x_1 = x_0 - \frac{F(x_0)}{F'(x_0)} = 2.8434$$

$$x_2 = 2.8434 - \frac{F(2.8434)}{F'(2.8434)}$$

$$= 2.7822$$

$$x_3 = 2.7822 - \frac{F(2.7822)}{F'(2.7822)}$$

$$= 2.7576$$

By, $x_4 = 2.7476$

$$x_5 = 2.7435$$

$$x_6 = 2.7418$$

$$x_7 = 2.7411$$

$$x_8 = 2.7408$$

$$x_9 = 2.7407$$

$$x_{10} = 2.7407$$

$\therefore$ The required root is 2.7407

HW find the +ve root of $2x^3 - 3x - 6 = 0$

Ans: 1.7838



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8. Find the -ve root of $x^3 - \sin x + 1 = 0$.

[radian mode]

Let $F(x) = x^3 - \sin x + 1$

$$F'(x) = 3x^2 - \cos x$$

$$F(0) = 1$$

$$F(-1) = 0.8415 \text{ (ve)}$$

$$F(-2) = -6.0907 \text{ (-ve)}$$

$\therefore$ The root lies between -1 & -2

Since $|F(-1)| < |F(-2)|$. Let us assume $x_0 = -1$

$$x_1 = x_0 - \frac{F(x_0)}{F'(x_0)}$$

$$x_1 = -1.3421$$

$$x_2 = -1.3421 - \frac{F(-1.3421)}{F'(-1.3421)}$$

$$= -1.2564$$

$$x_3 = -1.2491$$

$$x_4 = -1.2491$$

Since x_3 & x_4 are equal

$\therefore$ The required root is -1.2491