UNIT 3 SOLUTION OF EQUATIONS
INVERSE GAUSS JORDAN METHOD

Inverse Gauss Jordan Method

1. Using Gauss Jordan method find the inverse of the matrix $\begin{pmatrix} 2 & 2 & 3 \\ 2 & 1 & 1 \\ 1 & 3 & 5 \end{pmatrix}$

Let $A = \begin{pmatrix} 2 & 2 & 3 \\ 2 & 1 & 1 \\ 1 & 3 & 5 \end{pmatrix}$, $X = \begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{pmatrix}$ be the inverse of A

So that $AX = I$

The augmented matrix is,

$$\left[\begin{array}{ccc|ccc} 2 & 2 & 3 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 1 & 0 \\ 1 & 3 & 5 & 0 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 2 & 2 & 3 & 1 & 0 & 0 \\ 1 & -1 & -2 & -1 & 1 & 0 \\ 0 & 4 & 7 & -1 & 0 & 2 \end{array}\right] \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to 2R_3 - R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 2 & 2 & 3 & 1 & 0 & 0 \\ 0 & -1 & -2 & -1 & 1 & 0 \\ 0 & 0 & -1 & -5 & 4 & 2 \end{array}\right] R_3 \to R_3 + 4R_2$$

$$\sim \left[\begin{array}{ccc|ccc} 2 & 2 & 3 & 1 & 0 & 0 \\ 0 & -1 & 0 & 9 & -7 & -4 \\ 0 & 0 & -1 & -5 & 4 & 2 \end{array}\right] R_2 \to R_2 - 2R_3$$

$$\sim \begin{bmatrix} 2 & 0 & 3 & 19 & -14 & -8 \\ 0 & -1 & 0 & 9 & -7 & -4 \\ 0 & 0 & -1 & -5 & 4 & 2 \end{bmatrix} \quad R_1 \to R_1 + 2R_2$$

$$\sim \begin{bmatrix} 2 & 0 & 0 & 4 & -2 & -2 \\ 0 & -1 & 0 & 9 & -7 & -4 \\ 0 & 0 & -1 & -5 & 4 & 2 \end{bmatrix} \quad R_1 \to R_1 + 3R_3$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 2 & -1 & -1 \\ 0 & 1 & 0 & -9 & 7 & 4 \\ 0 & 0 & 1 & 5 & -4 & -2 \end{bmatrix} \quad \begin{matrix} R_1 \to \frac{R_1}{2} \\ R_2 \to (-1) \times R_2 \\ R_3 \to (-1) \times R_3 \end{matrix}$$

Hence the inverse of the given matrix A is

$$\begin{bmatrix} 2 & -1 & -1 \\ -9 & 7 & 4 \\ 5 & -4 & -2 \end{bmatrix}$$

2) Find the inverse of $\begin{pmatrix} 2 & 2 & 6 \\ 2 & 6 & -6 \\ 4 & -8 & -8 \end{pmatrix}$ using gauss Jordan method.

Let $A = \begin{pmatrix} 2 & 2 & 6 \\ 2 & 6 & -6 \\ 4 & -8 & -8 \end{pmatrix}$ and

$X = \begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{pmatrix}$ be the inverse of A

so that $AX = I$

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The Augmented matrix is

$$\begin{bmatrix} 2 & 2 & 6 & 1 & 0 & 0 \\ 2 & 6 & -6 & 0 & 1 & 0 \\ 4 & -8 & -8 & 0 & 0 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 2 & 2 & 6 & 1 & 0 & 0 \\ 0 & 4 & -12 & -1 & 1 & 0 \\ 0 & -12 & -20 & -2 & 0 & 1 \end{bmatrix} \begin{matrix} \\ R_2 \to R_2 - R_1 \\ R_3 \to R_3 - 2R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 2 & 2 & 6 & 1 & 0 & 0 \\ 0 & 4 & -12 & -1 & 1 & 0 \\ 0 & 0 & -56 & -5 & 3 & 1 \end{bmatrix} R_3 \to R_3 + 3R_2$$

$$\sim \begin{bmatrix} 1 & 1 & 3 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & -3 & \frac{-1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & 1 & \frac{5}{56} & \frac{-3}{56} & \frac{-1}{56} \end{bmatrix} \begin{matrix} R_1 \to R_1 \div 2 \\ R_2 \to R_2 \div 4 \\ R_3 \to R_3 \div (-56) \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 0 & \frac{13}{56} & \frac{9}{56} & \frac{3}{56} \\ 0 & 1 & 0 & \frac{1}{56} & \frac{5}{56} & \frac{-3}{56} \\ 0 & 0 & 1 & \frac{5}{56} & \frac{-3}{56} & \frac{-1}{56} \end{bmatrix} \begin{matrix} R_1 \to R_1 - 3R_3 \\ R_2 \to R_2 + 3R_3 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & \frac{12}{56} & \frac{4}{56} & \frac{6}{56} \\ 0 & 1 & 0 & \frac{1}{56} & \frac{5}{56} & \frac{-3}{56} \\ 0 & 0 & 1 & \frac{5}{56} & \frac{-3}{56} & \frac{-1}{56} \end{bmatrix}$$

∴ The inverse of the given matrix A is

$$\begin{bmatrix} \frac{12}{56} & \frac{4}{56} & \frac{6}{56} \\ \frac{1}{56} & \frac{5}{56} & \frac{-3}{56} \\ \frac{5}{56} & \frac{-3}{56} & \frac{-1}{56} \end{bmatrix} = \frac{1}{56} \begin{bmatrix} 12 & 4 & 6 \\ 1 & 5 & -3 \\ 5 & -3 & -1 \end{bmatrix}$$