UNIT 3 SOLUTION OF EQUATIONS
GAUSS JORDAN METHOD

Gauss Jordan Method

In Gauss Jordan method, the coefficient matrix is reduced to a diagonal matrix (or even a unit matrix) rather than a triangular matrix as in the Gaussian method. Here the elimination of the unknowns is done not only in the equation below but also in the eqns above the leading diagonal. Here we get the solution without using the back substitution method.

1. Solve by Gauss Jordan method

$$10x+y+z=12$$
$$2x+10y+z=13$$
$$x+y+5z=7$$

Augmented matrix form

$$\begin{pmatrix} 10 & 1 & 1 & 12 \\ 2 & 10 & 1 & 13 \\ 1 & 1 & 5 & 7 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 1/10 & \frac{1}{10} & \frac{12}{10} \\ 2 & 10 & 1 & 13 \\ 1 & 1 & 5 & 7 \end{pmatrix} R_1 \to \frac{R_1}{10}$$

$$\sim \begin{pmatrix} 1 & \frac{1}{10} & \frac{1}{10} & \frac{12}{10} \\ 0 & \frac{49}{5} & \frac{4}{5} & \frac{53}{5} \\ 0 & \frac{9}{10} & \frac{49}{10} & \frac{58}{10} \end{pmatrix} \begin{array}{l} R_2 \to R_2-2R_1 \\ R_3 \to R_3-R_1 \end{array}$$

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$$\sim \begin{pmatrix} 1 & \frac{1}{10} & \frac{1}{10} & \frac{12}{10} \\ 0 & 1 & \frac{4}{49} & \frac{53}{49} \\ 0 & \frac{9}{10} & \frac{49}{10} & \frac{58}{10} \end{pmatrix} R_2 \to R_2 \div \frac{49}{5}$$

$$\sim \begin{pmatrix} 1 & \frac{1}{10} & \frac{1}{10} & \frac{12}{10} \\ 0 & 1 & \frac{4}{49} & \frac{53}{49} \\ 0 & 0 & 4.8266 & 4.8266 \end{pmatrix} R_3 \to R_3 - \frac{9}{10} R_2$$

$$\sim \begin{pmatrix} 1 & \frac{1}{10} & \frac{1}{10} & \frac{12}{10} \\ 0 & 1 & \frac{4}{49} & \frac{53}{49} \\ 0 & 0 & 1 & 1 \end{pmatrix} R_3 \to \frac{R_3}{4.8266}$$

$$\sim \begin{pmatrix} 1 & \frac{1}{10} & 0 & \frac{11}{10} \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix} \begin{matrix} R_1 \to R_1 - \frac{1}{10} R_3 \\ R_2 \to R_2 - \frac{4}{49} R_3 \end{matrix}$$

$$\sim \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix} R_1 \to R_1 - \frac{1}{10} R_2$$

$$\therefore \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\Rightarrow x = 1, \; y = 1, \; z = 1$$

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Procedure :-

1. Write the augmented matrix

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{pmatrix}$$ for the given system of equations

2. Using elementary row transformation reduce the given matrix into a diagonal matrix say

$$\begin{pmatrix} c_{11} & 0 & 0 & d_1 \\ 0 & c_{22} & 0 & d_2 \\ 0 & 0 & c_{33} & d_3 \end{pmatrix}$$

3) From the above matrix we can find the values of x, y and z.

Q. Solve $x+3y+3z=16$, $x+4y+3z=18$, $x+3y+4z=19$ by Gauss-Jordan Method

Given:
$$x+3y+3z=16$$
$$x+4y+3z=18$$
$$x+3y+4z=19$$

The Augmented matrix is

$$\begin{pmatrix} 1 & 3 & 3 & 16 \\ 1 & 4 & 3 & 18 \\ 1 & 3 & 4 & 19 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 3 & 3 & 16 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{pmatrix} \begin{matrix} \\ R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

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$$\begin{pmatrix} 1 & 0 & 3 & 10 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{pmatrix} R_1 \to R_1 - 3R_2$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{pmatrix} R_1 \to R_1 - 3R_3.$$

The matrix finally reduces to the form given by

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$\therefore\ x=1,\ y=2,\ z=3.$