UNIT 3 SOLUTION OF EQUATIONS
GAUSS SEIDAL METHOD

Iterative Methods:

i) Gauss-Jacobi method

ii) Gauss Seidel Method

Gauss Seidal Method:-

Let the system of simultaneous equation be

$$\left.\begin{aligned} a_{11}x_1 + b_{12}y + c_{13}z &= d_1 \\ a_{21}x + b_{22}y + c_{23}z &= d_2 \\ a_{31}x + b_{32}y + c_{33}z &= d_3 \end{aligned}\right\} \rightarrow (1)$$

The above system can be written as

$$x_1 = \frac{1}{a_{11}}(d_1 - a_{12}y_2 - a_{13}z_3) \rightarrow (2)$$

$$y_1 = \frac{1}{a_{22}}(d_2 - a_{21}x_1 - a_{23}z_3) \rightarrow (3)$$

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$$x_3 = \frac{1}{a_{33}}(d_3 - a_{31}x - a_{32}y) \rightarrow (4)$$

3) Substituting $y=0$ and $z=0$ in (1), we get the value for x and it is denoted by x_1:

Substituting $x = x_1$ and $z = 0$ we get the value for y. and it is denoted by y_1. Substituting $x = x_1$ and $y = y_1$ in 3 we get the value for z and it is denoted by z_1. These values of x_1, y_1, z_1 are called first iterative values of x, y and z.

4. The above process may continue for second, third, fourth, etc. iterations.

1) Solve by Gauss Seidel method

$$x + y + 54z = 110$$
$$27x + 6y - 5z = 85$$
$$6x + 15y + 2z = 72$$

Let us rearrange the equations

$$27x + 6y - 5z = 85 \rightarrow (1)$$
$$6x + 15y + 2z = 72 \rightarrow (2)$$
$$x + y + 54z = 110 \rightarrow (3)$$

$\therefore |27| > |6| + |5|$
$|15| > |6| + |2|$
$|54| > |1| + |1|$

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$$① \Rightarrow x = \frac{85 - 6y + 5z}{27}$$

$$② \Rightarrow y = \frac{72 - 6x - 2z}{15}$$

$$③ \Rightarrow z = \frac{110 - x - y}{54}$$

Let $y_0 = z_0 = 0$.

$x = \frac{1}{27}[85 - 6y + 5z]$	$y = \frac{1}{15}[72 - 6x - 2z]$	$z = \frac{1}{54}[110 - x - y]$
$x_1 = \frac{85}{27} = 3.148$	$y_1 = 3.5408$	$z_1 = 1.913$
$x_2 = 2.715$	$y_2 = 3.458$	$z_2 = 1.923$
$x_3 = 2.735$	$y_3 = 3.449$	$z_3 = 1.923$
$x_4 = 2.738$	$y_4 = 3.4484$	$z_4 = 1.9224$
$x_5 = 2.738$	$y_5 = 3.4484$	$z_5 = 1.9224$

$\therefore x = 2.738$

$y = 3.4484$

$z = 1.9224$

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2. Solve the following System by Gauss Seidal

method: $9x - y + 2z = 9$

$x + 10y - 2z = 15$

$2x - 2y - 13z = -17$

The given System of eqns are

$9x - y + 2z = 9 \rightarrow (1)$

$x + 10y - 2z = 15 \rightarrow (2)$

$2x - 2y - 13z = -17 \rightarrow (3)$

Clearly the coeff matrix is diagonally dominant, So we can apply Gauss seidal method

$x = \frac{1}{9}[9 + y - 2z]$

$y = \frac{1}{10}[15 - x + 2z]$

$z = \frac{1}{13}[17 + 2x - 2y]$

Let $y_0 = z_0 = 0$.

$x = \frac{1}{9}[9+y-2z]$	$y = \frac{1}{10}[15-x+2z]$	$z = \frac{1}{13}[17+2x-2y]$
$x_1 = 1$	$y_1 = 1.4$	$z_1 = 1.246$
$x_2 = 0.8786$	$y_2 = 1.6613$	$z_2 = 1.1872$
$x_3 = 0.9208$	$y_3 = 1.6454$	$z_3 = 1.1962$

UNIT 3 SOLUTION OF EQUATIONS
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$x_4 = 0.9170$	$y_4 = 1.6475$	$z_4 = 1.1952$
$x_5 = 0.9174$	$y_5 = 1.6473$	$z_5 = 1.1954$
$x_6 = 0.9174$	$y_6 = 1.6473$	$z_6 = 1.1954$

$\therefore x = 0.9174, y = 1.6473, z = 1.1954$