CLOSURE OF A SET OF FD's

The closure of a set of functional dependencies (F+) is the set of all functional dependencies that can be derived from the given set using Armstrong's axioms (reflexivity, augmentation, and transitivity).

If "F" is a functional dependency then closure of functional dependency can be denoted using "{F} +".

RULES

- Step-1 : Add the attributes which are present on Left Hand Side in the original functional dependency.
- Step-2 : Now, add the attributes present on the Right Hand Side of the functional dependency.
- Step-3 : With the help of attributes present on Right Hand Side, check the other attributes that can be derived from the other given functional dependencies. Repeat this process until all the possible attributes which can be derived are added in the closure.

SUPER KEY AND CANDIDATE KEY

- Super key – Set of all attributes whose closure contains all attributes of given relation
- Candidate Key –minimal set of attributes whose attribute closure is set of all attributes of relation

CLOSURE OF A SET OF FUNCTIONAL DEPENDENCIES

For a set X of attributes, we call the closure of X (with respect to a set of functional dependencies F), noted X+, the maximum set of attributes such that X→X+ (as a consequence of F)

Example: Consider the relation scheme R(A,B,C,D) with functional dependencies {A}→{C} and {B}→{D}.

- {A}+ = {A,C}

▪{B}+ = {B,D}

▪{C}+={C}

▪{D}+={D}

▪{A,B}+ = {A,B,C,D} {A,B} is a superkey because:

▪It determines all attributes

CLOSURE PROPERTY

Example : R = (A, B, C, G, H, I)

F = { A → B, A → C ,CG → H , CG → I , B → H}

Find some members of F+

R = (A, B, C, G, H, I)

F = { A → B, A → C ,CG → H , CG → I , B → H}

- some members of F+
  - A → H (by transitivity from A → B and B → H)
  - AG → I (A →C /*Augmentation rule X → Y; XZ →YZ */

    AG → CG

    CG → I given /* Transitivity rule X → Y; XZ →YZ */

    AG →I
- by augmenting A → C with G, to get AG → CG and then transitivity with CG → I – CG → HI
- By Secondary rule Union.

We can further simplify manual computation of F+ by using the following additional rules.

- If → holds and → holds, then → holds (union)
- If → holds, then → holds and → holds (decomposition)
- If → holds and → holds, then → holds (pseudotransitivity)

- The above rules can be inferred from Armstrong's axioms.

EXAMPLE 1

Consider a relation R(A,B,C,D,E) having below mentioned functional dependencies.

1. **FD1 : A → B**
2. **FD2 : B → C**
3. **FD2 : C → D**
4. **FD3 : D → E**

1. $\{A\}^+ = \{A, B, C, D, E\}$
2. $\{B\}^+ = \{B, C, D, E\}$
3. $\{C\}^+ = \{C, D, E\}$
4. $\{D\}^+ = \{D, E\}$
5. $\{E\}^+ = \{E\}$

1. $\{AD\}^+ = \{A, D, B, C, E\}$
2. $\{CD\}^+ = \{C, D, E\}$

EXAMPLE 2

Consider the table student_details having (Roll_No, Name,Marks, Location) as the attributes and having two functional dependencies. calculate the closure of all the attributes present in the relation

1. **FD1 : Roll_No → Name, Marks**
2. **FD2 : Name → Marks, Location**

{Roll_no}+ = {Roll_No, Marks, Name, Location}

{Name}+ = {Name, Marks, Location}

{Marks}+ = {Marks}

{Location}+ = { Location}

EXAMPLE 3

Consider a relation R(A,B,C,D,E) having below mentioned functional dependencies. Calculate the candidate key.

1. **FD1 : A →BC**
2. **FD2 : C →B**
3. **FD3 : D →E**
4. **FD4 : E →D**

1. $\{A\}^+ = \{A, B, C\}$
2. $\{B\}^+ = \{B\}$
3. $\{C\}^+ = \{B, C\}$
4. $\{D\}^+ = \{D, E\}$
5. $\{E\}^+ = \{E\}$

A single attribute is unable to determine all the attribute.

Here, we need to combine two or more attributes to determine the candidate keys.

$\{A, D\}^+ = \{A, B, C, D, E\}$

$\{A, E\}^+ = \{A, B, C, D, E\}$