



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore-641035.



Homogeneous differential equations with variable coefficient.

Euler form Legendre's form.

sachy's form $(ax+b)$ form.

An equation of the form $a_0x^n D^n + a_1x^{n-1}D^{n-1} + \dots + a_{n-1}D + a_n y = 0$, where a_0, a_1, \dots, a_n are constant and y is a function is called Euler homogeneous linear eqn.

Rule:

$$x = e^t \text{ (or) } t = \log(x), \quad y = v(t), \quad y' = v'(t) \cdot \frac{dt}{dx}, \quad y'' = v''(t) \cdot \frac{d^2t}{dx^2} + v'(t) \cdot \frac{d^2t}{dx^2}$$

$$\frac{x dy}{dx} = x D y = \theta y, \quad \theta = \frac{dt}{dx}$$

$$x^2 \frac{d^2 y}{dx^2} = x^2 D^2 y = \theta(\theta-1)y$$



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UNIT-II ORDINARY DIFFERENTIAL EQUATIONS

Homo.Lin.Eqns.of Euler's type

Solve $x^2 \frac{d^2y}{dx^2} + 4x \frac{dy}{dx} + 2y = x^2 + \frac{1}{x^2}$

$$x^2 \frac{d^2y}{dx^2} + 4x \frac{dy}{dx} + 2y = x^2 + \frac{1}{x^2}$$

$$x^2 D^2 y + 4x D y + 2y = x^2 + \frac{1}{x^2}$$

$$(x^2 D^2 + 4x D + 2)y = x^2 + \frac{1}{x^2}$$

$$x = e^t \quad t = \log x \quad R(x) = x^2 + \frac{1}{x^2}$$

$$x^2 D^2 y = \theta(\theta-1)$$

$$\therefore = \theta^2 - \theta$$

$$x D y = \theta$$

$$(\theta^2 - \theta + 4\theta + 2)y = e^{2t} + e^{-2t}$$

$$(\theta^2 + 3\theta + 2)y = e^{2t} + e^{-2t}$$

$$\text{The A.E is } m^2 + 3m + 2 = 0 \quad [m+1] [m+2] = 0$$

$$(m+1)(m+2) = 0$$

$$m = -1, m = -2$$

$$\text{The C.F is } Ae^{-x} + Be^{-2x} = y(x + Cx + x^2)$$

$$P.I_1 = \frac{e^{2t}}{\theta^2 + 3\theta + 2} = \frac{e^{2t}}{(2+1)^2 + 3(2+1) + 2} = \frac{e^{2t}}{(-2)^2 + 3(-2) + 2} = \frac{e^{2t}}{4 - 6 + 2} = \frac{e^{2t}}{-2} = -\frac{1}{2}e^{2t}$$

$$P.I_2 = \frac{1}{\theta^2 + 3\theta + 2} e^{-2t} = \frac{1}{(-2)^2 + 3(-2) + 2} e^{-2t} = \frac{1}{4 - 6 + 2} e^{-2t} = \frac{1}{-2} e^{-2t} = -\frac{1}{2} e^{-2t}$$

$$= \frac{1}{(-2)^2 + 3(-2) + 2} e^{-2t} = \frac{1}{4 - 6 + 2} e^{-2t} = \frac{1}{-2} e^{-2t} = -\frac{1}{2} e^{-2t}$$

$$= \frac{t}{2} e^{-2t} + C e^{-2t}$$

$$= \frac{t}{2} e^{-2t} + C e^{-2t}$$



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$$= \frac{t}{-1} e^{-2t} \Rightarrow -te^{-2t}$$

$$P.I = P.I_1 + P.I_2.$$

$$= \frac{1}{2} e^{2t} - te^{-2t}$$

$$= \frac{(et)^2}{12} - \frac{t}{(et)^2}$$

$$P.I = \frac{x^2}{12} - \frac{\log x}{x^2}$$

$$Y = C.F + P.I$$

$$= Ae^{-t} + Be^{-2t} + \frac{x^2}{12} - \frac{\log x}{x^2}$$