Homogeneous differential equations with variable coefficient.

Euler form or cauchy's form Legender's form. (ax+b) form.

An equation of the form $(a_0 x^n D^n + a_1 x^{n-1} D^{n-1} + \dots + a_{n-1} xD + a_n) y = x$. where $a_0, a_1 \dots a_n$ are constant and 'x' is a function is called euler homogeneous linear eqn.

Rule:

$$x = e^t \text{ (or) } t = \log x$$

$$x\frac{dy}{dx} = xDy = \theta y$$

$$x^2\frac{d^2y}{dx^2} = x^2D^2y = \theta(\theta-1)y.$$

Solve $x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = x^2 + \frac{1}{x^2}$.

$x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = x^2 + \frac{1}{x^2}$.

$x^2D^2y + 4xDy + 2y = x^2 + \frac{1}{x^2}$.

$(x^2D^2 + 4xD + 2)y = x^2 + \frac{1}{x^2}$.

$x = e^t$ (or) $t = \log x$ $\qquad R(x) = x^2 + \frac{1}{x^2}$

$x^2D^2y = \theta(\theta-1)$ $\qquad = (e^t)^2 + \frac{1}{e^{t^2}}$

$= \theta^2 - \theta$

$xDy = \theta$. $\qquad R(x) = e^{2t} + e^{-2t}$.

$((\theta^2-\theta) + 4\theta + 2)y = e^{2t} + e^{-2t}$.

$(\theta^2 + 3\theta + 2)y = e^{2t} + e^{-2t}$.

The A.E is $m^2 + 3m + 2 = 0$

$(m+1)(m+2) = 0$

$m = -1, m = -2$.

The C.F is $Ae^{-x} + Be^{-2x}$.

$P.I_1 = \frac{1}{\theta^2 + 3\theta + 2} e^{2t}$ $[\theta = a = 2]$.

$= \frac{1}{(2)^2 + 3(2) + 2}$

$P.I_1 = \frac{1}{12} e^{2t}$.

$P.I_2 = \frac{1}{\theta^2 + 3\theta + 2} e^{-2t}$ $[\theta = a = 2]$.

$= \frac{1}{(-2)^2 + 3(-2) + 2}$ $\qquad = \frac{t}{2\theta + 3} e^{-2t}$

$$= \frac{t}{-1} e^{-2t} \Rightarrow -te^{-2t}.$$

$$P.I = P.I_1 + P.I_2.$$

$$= \frac{1}{2} e^{2t} - te^{-2t}.$$

$$= \frac{(e^t)^2}{12} - \frac{t}{(e^t)^2}.$$

$$P.I = \frac{x^2}{12} - \frac{\log x}{x^2}.$$

$$y = C.F + P.I$$

$$= Ae^{-x} + Be^{-2x} + \frac{x^2}{12} - \frac{\log x}{x^2}.$$