



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore-641035.



UNIT-II ORDINARY DIFFERENTIAL EQUATIONS

Homo.Lin.Eqns.of legendre's type

Legendre's Linear differential equations:

An eqn which is of the form $(ax+b)^n \frac{d^n y}{dx^n} + a_1 (ax+b)^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + \dots + a_{n-1} (ax+b) \frac{dy}{dx} + a_n y = R(x)$.

is called L.I.D.E.

Rule:

Let $e^t = (ax+b)$ or $t = \log(ax+b)$.

$$(ax+b)D = a\theta.$$

$$(ax+b)^2 D^2 = a^2 (\theta^2) \quad \theta(\theta-1).$$

Prob:

$$\text{Solve } (2x+3)^2 y'' - (2x+3)y' - 12y = 6x.$$

Sln:

$$(2x+3) \frac{d^2 y}{dx^2} - (2x+3) \frac{dy}{dx} - 12y = 6x.$$

$$[(2x+3)^2 D^2 - (2x+3)D - 12]y = 6x.$$

$$e^t = 2x+3 \quad t = \log(2x+3). \quad R(x) = 6x$$

$$R = \frac{e^{t-3}}{2}$$

$$R(x) = \frac{1}{2} \left(\frac{e^{t-3}}{2} \right)$$

$$(2x+3)D = 2\theta$$

$$(2x+3)^2 D^2 = 4(\theta^2 - \theta).$$

$$4(\theta^2 - \theta) - 2\theta - 12)y = 3e^{t-3}.$$

$$4\theta^2 - 6\theta - 12)y = 3e^{t-3}.$$

$$\text{The A.E } 4m^2 - 6m - 12 = 0.$$

$$4m^2 - 3m - 6 = 0.$$



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore-641035.



UNIT-II ORDINARY DIFFERENTIAL EQUATIONS

Homo.Lin.Eqns.of legendre's type

$$\begin{aligned}
 m &= -\frac{b \pm \sqrt{b^2 - 4ac}}{2a} \\
 a = 2, b = -3, c = -6 \\
 m &= \frac{3 \pm \sqrt{9 + 48}}{4} = \frac{3 \pm \sqrt{57}}{4} \\
 m &= \frac{3 + \sqrt{57}}{4}, \quad m_2 = \frac{3 - \sqrt{57}}{4} \\
 m_1 &= \frac{3 + \sqrt{57}}{4}, \quad m_2 = \frac{3 - \sqrt{57}}{4} \\
 \text{The C.F.} &= Ae^{\frac{3+\sqrt{57}}{4}x} + Be^{\frac{3-\sqrt{57}}{4}x} \\
 P.I. &= \frac{1}{40^2 - 60 - 12} \cdot 3e^t - 9. \\
 P.I. &= \frac{3}{40^2 - 60 - 12} e^t = \frac{3}{14} e^t. \\
 P.I. &= \frac{1}{40^2 - 60 - 12} q e^{ot} = \frac{1}{12} q e^{ot}. \\
 P.I. &= P.I. + P.I. \\
 P.I. &= \frac{3}{14} e^t + \frac{9}{12} e^{ot} = \frac{3}{14} e^t + \frac{3}{4} e^{ot}. \\
 P.I. &= \frac{3}{14} e^{\log(2x+3)} + \frac{9}{12} e^{ot} = \frac{3}{14} e^{\log(2x+3)} + \frac{3}{4} e^{ot} = \frac{3}{14} (2x+3) + \frac{3}{4} e^{ot}. \\
 P.I. &= \frac{9}{12} - \frac{3}{14} (2x+3). \\
 Y &= C.F. + P.I. \\
 &= Ae^{\frac{3+\sqrt{57}}{4}x} + Be^{\frac{3-\sqrt{57}}{4}x} + \frac{9}{12} - \frac{3}{14} (2x+3).
 \end{aligned}$$