



Legendre's Linear differential equations:

An eqn which is of the form  $(ax+b)^n \frac{d^n y}{dx^n} + a_1 (ax+b)^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + \dots + a_{n-1} (ax+b) \frac{dy}{dx} + a_n y = R(x)$ .

is called L.I.D.E.

Rule:

Let  $e^t = (ax+b)$  or  $t = \log(ax+b)$ .

$$(ax+b)D = a\theta.$$

$$(ax+b)^2 D^2 = a^2 (\theta^2) \quad \theta(\theta-1).$$

Prob:

$$\text{Solve } (2x+3)^2 y'' - (2x+3)y' - 12y = 6x.$$

Sln:

$$(2x+3) \frac{d^2 y}{dx^2} - (2x+3) \frac{dy}{dx} - 12y = 6x.$$

$$[(2x+3)^2 D^2 - (2x+3)D - 12]y = 6x.$$

$$e^t = 2x+3 \quad t = \log(2x+3). \quad R(x) = 6x$$

$$x = \frac{e^t - 3}{2}$$

$$R(x) = 6 \left( \frac{e^t - 3}{2} \right)$$

$$(2x+3)D = 2\theta \quad [(x \text{ pol}) \text{ and } (R \text{ pol}) \text{ for } x \in \mathbb{C}] \quad R(x) = 3e^t - 9.$$

$$(2x+3)^2 D^2 = 4(\theta^2 - \theta).$$

$$4(\theta^2 - \theta) - 2\theta - 12)y = 3e^t - 9. \quad [(\text{R pol}) \text{ and } (x \text{ pol}) \text{ for } x \in \mathbb{C}] \quad 4\theta^2 - 6\theta - 12)y = 3e^t - 9.$$

$$\text{The A.E } 4m^2 - 6m - 12 = 0.$$

$$4m^2 - 3m - 6 = 0.$$



# SNS COLLEGE OF TECHNOLOGY

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## UNIT-II ORDINARY DIFFERENTIAL EQUATIONS

Homo.Lin.Eqns.of legendre's type

$$\begin{aligned}
 m &= -\frac{b \pm \sqrt{b^2 - 4ac}}{2a} \\
 a = 2, b = -3, c = -6 &\quad m_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{3 + \sqrt{9 + 48}}{4} = \frac{3 + \sqrt{57}}{4} x + C_1 e^{(2x)} \\
 m &= \frac{3 \pm \sqrt{9+48}}{4} = \frac{3 \pm \sqrt{57}}{4} x + C_2 e^{(2x)} \\
 m &= \frac{3 \pm \sqrt{57}}{4} x + C_3 e^{(2x)} \\
 m_1 &= \frac{3 + \sqrt{57}}{4} x + C_1 e^{(2x)} \\
 \text{The C.F.} &= A e^{\frac{3+\sqrt{57}}{4} x} + B e^{\frac{3-\sqrt{57}}{4} x} \\
 P.I. &= \frac{1}{40^2 - 60 - 12} = \frac{1}{32} e^{3x} - 9. \\
 P.I. &= \frac{1}{40^2 - 60 - 12} e^t = \frac{1}{32} e^{3t} - 9. \\
 &= \frac{3}{-14} e^{3t}. \\
 P.I. &= \frac{1}{40^2 - 60 - 12} q e^{ot} = \frac{1}{32} e^{3t} - 9. \\
 &= \frac{1}{-12} q e^{3t}. \\
 P.I. &= P.I. + P.I. \\
 P.I. &= \frac{3}{-14} e^{3t} + \frac{9}{12} e^{3t} = \frac{1}{14} e^{3t} - 9. \\
 P.I. &= \frac{3}{-14} e^{\log(2x+3)} + \frac{9}{12} e^{3t} = \frac{1}{14} e^{3t} - 9. \\
 P.I. &= \frac{9}{12} - \frac{3}{14} (2x+3). \\
 Y &= C.F. + P.I. \\
 &= A e^{\frac{3+\sqrt{57}}{4} x} + B e^{\frac{3-\sqrt{57}}{4} x} + \frac{9}{12} - \frac{3}{14} (2x+3).
 \end{aligned}$$