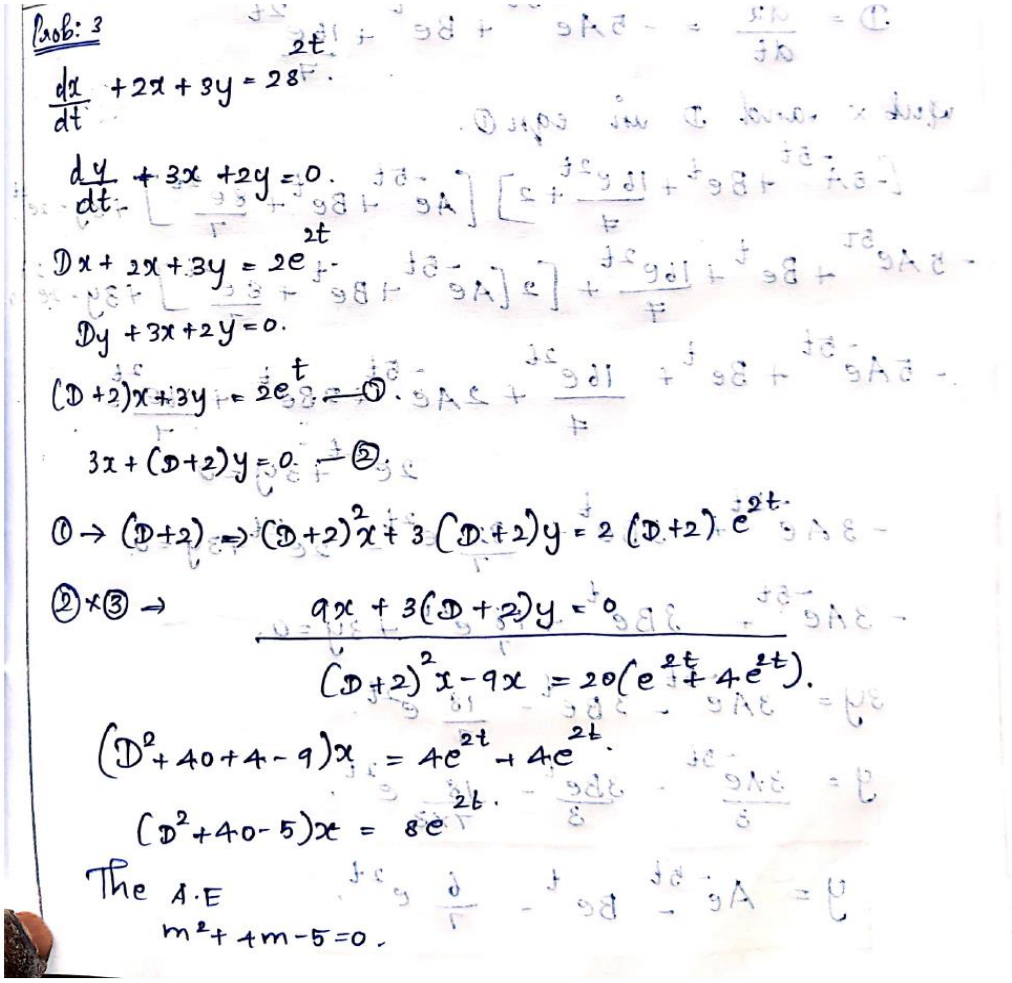
UNIT-II ORDINARY DIFFERENTIAL EQUATIONS SIMULTANIOUS FIRST ORDER LINEAR DIFF. EQUATIONS

Solve the below simultaneous linear differential equations for problem 3.



Prob: 3

$\frac{dx}{dt} + 2x + 3y = 2e^{2t}$.

$\frac{dy}{dt} + 3x + 2y = 0$.

$Dx + 2x + 3y = 2e^{2t}$

$Dy + 3x + 2y = 0$.

$(D+2)x + 3y = 2e^{t} \rightarrow (1)$.

$3x + (D+2)y = 0 \rightarrow (2)$.

$(1) \rightarrow (D+2) \Rightarrow (D+2)^2 x + 3(D+2)y = 2(D+2)e^{2t}$.

$(2) \times (3) \rightarrow \quad 9x + 3(D+2)y = 0$

$(D+2)^2 x - 9x = 2e^{2t}(e^{2t} + 4e^{2t})$.

$(D^2 + 4D + 4 - 9)x = 4e^{2t} + 4e^{2t}$.

$(D^2 + 4D - 5)x = 8e^{2t}$.

The A.E

$m^2 + 4m - 5 = 0$.

$(m+5)(m-1)=0.$

$m=-5, m=1$

$C.F = Ae^{-5t} + Be^{t}.$

$P.I = \frac{1}{D^2+4D-5} 8e^{2t}$ $[D=a=2]$

$= \frac{1}{7} 8e^{2t}.$

$P.I = \frac{8}{7} e^{2t}.$

$x = C.F + P.I.$

$x = Ae^{-5t} + Be^{t} + \frac{8}{7} e^{2t}.$

$D = \frac{dx}{dt} = -5Ae^{-5t} + Be^{t} + \frac{16e^{2t}}{7}$

Sub x and D in eqn ①.

$\left[-5Ae^{-5t} + Be^{t} + \frac{16e^{2t}}{7} + 2\right]\left[Ae^{-5t} + Be^{t} + \frac{8e^{2t}}{7}\right] + 3y = 2e^{2t}.$

$-5Ae^{5t} + Be^{t} + \frac{16e^{2t}}{7} + \left[2\left[Ae^{-5t} + Be^{t} + \frac{8e^{2t}}{7}\right] + 3y - 2e^{2t} = 0\right.$

$-5Ae^{-5t} + Be^{t} + \frac{16e^{2t}}{7} + 2Ae^{-5t} + 2Be^{t} + \frac{16e^{2t}}{7}$

$2e^{2t} + 3y = 0$

$-3Ae^{-5t} + 3Be^{t} + \frac{32}{7} e^{2t} - 2e^{2t} + 3y = 0.$

$-3Ae^{-5t} + 3Be^{t} + \frac{18}{7} e^{2t} + 3y = 0.$

$3y = 3Ae^{-5t} - 3Be^{t} - \frac{18}{7} e^{2t}.$

$y = \frac{3Ae^{-5t}}{3} - \frac{3Be^{t}}{3} - \frac{18}{7\times 3} e^{2t}.$

$$\boxed{y = Ae^{-5t} - Be^{t} - \frac{6}{7} e^{2t}.}$$