UNIT 3 PARTIAL DIFFERENTIAL EQUATIONS
Solutions of standard types of first order partial differential equations

Solution of Standard types of first order PDE

A partial differential equation in which the partial derivative occurring are of the first degree is said to be linear, otherwise it is said to be non-linear.

Standard types:

Type 1 : $F(p,q)=0$

Type 2 : $z = px + qy + f(p,q)$ [Clairaut's form]

Type 3 : $f(z,p,q)=0$

Type 4 : $f_1(x,p) = f_2(y,q)$

Type 1 : Working Rule:

1. Let $z = ax+by+c$ be the complete integral. $p=\frac{\partial z}{\partial x}=a$, $q=\frac{\partial z}{\partial y}=b$
2. put $b=\phi(a)$ for general solution
3. There is no singular integral

1. Solve $p+q=pq$

Soln: $p+q=pq \rightarrow (1)$

Let $z = ax+by+c \rightarrow (2)$

Complete Integral:

Diff partially wrt 'x' and y

$\frac{\partial z}{\partial x} = a$, $p=a$ | $\frac{\partial z}{\partial y} = b$, $q=b$

Sub the above values in (1) we get

$a+b=ab$

$a=ab-b$ $\quad a=b(a-1) = b=\frac{a}{a-1}$

The complete Integral is,

$$z = ax + \left(\frac{a}{a-1}\right)y + c \rightarrow (3)$$

Singular Integral:

diff (3) p wrt 'a' and 'c' and equal to zero

$$\frac{\partial z}{\partial a} = x + \left[\frac{(a-1)(1)-a(1)}{(a-1)^2}\right]y = 0, \quad \frac{\partial z}{\partial c} = 1 \neq 0.$$

There is no singular Integral