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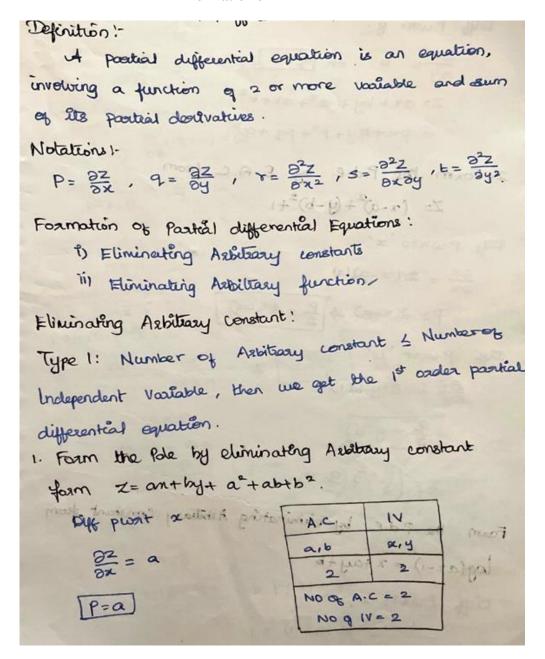*(An Autonomous Institution)*
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UNIT 3 PARTIAL DIFFERENTIAL EQUATIONS
Formation of PDE

Definition:-

A partial differential equation is an equation, involving a function q 2 or more variable and sum of its partial derivatives.

Notations:-

$$P = \frac{\partial z}{\partial x}, \quad q = \frac{\partial z}{\partial y}, \quad r = \frac{\partial^2 z}{\partial x^2}, \quad s = \frac{\partial^2 z}{\partial x \partial y}, \quad t = \frac{\partial^2 z}{\partial y^2}.$$

Formation of Partial differential Equations :

   i) Eliminating Arbitrary constants

   ii) Eliminating Arbitrary function

Eliminating Arbitrary Constant :

Type 1: Number of Arbitrary constant ≤ Number of Independent Variable, then we get the 1st order partial differential equation.

1. Form the Pde by eliminating Arbitrary constant form $z = ax + by + a^2 + ab + b^2$.

Diff p.w.r.t x with partially

$$\frac{\partial z}{\partial x} = a$$

$$\boxed{P = a}$$

A.C	IV
a, b	x, y
2	2
NO of A.C = 2	
No q IV = 2	

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Diff P w.r.t y.

$$\frac{\partial z}{\partial y} = b \quad \Rightarrow \boxed{q = b}$$

$$Z = ax + by + a^2 + ab + b^2.$$

$$= px + qy + p^2 + pq + q^2.$$

2. Form the P.d.e by E.A.C from

$$Z = (x-a)^2 + (y-b)^2 + 1$$

Diff P w.r.to 'x'.

$$\frac{\partial z}{\partial x} = 2(x-a)(1)$$

$$P = 2(x-a) \Rightarrow \boxed{\frac{P}{2} = (x-a)}$$

Diff P w.r.t 'y'

$$\frac{\partial z}{\partial y} = 2(y-b)(1)$$

$$\boxed{\frac{q}{2} = y-b}$$

$$z = \left(\frac{P}{2}\right)^2 + \left(\frac{q}{2}\right)^2 + 1.$$

Form the P.d.e by eliminating Arbitrary Constant from

$$\log(az-1) = x + ay + b$$

Diff P w.r.to x,

$$\frac{1}{(az-1)} \frac{\partial}{\partial x}(az-1) = 1$$

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$$\frac{1}{az-1} \; a \; \frac{\partial z}{\partial x} = 1$$

$$\frac{ap}{az-1} = 1 \qquad \longrightarrow \textcircled{1}.$$

Diff P w.r.t y,

$$\frac{1}{(az-1)} \; \frac{\partial}{\partial y}(az-1) = a$$

$$\frac{1}{(az-1)} \; a \; \frac{\partial z}{\partial y} = a$$

$$\frac{aq}{az-1} = a$$

$$\frac{q^2}{az-1} = 1 \qquad \longrightarrow \textcircled{2}.$$

Equating ① & ②.

$$\frac{ap}{az-1} = \frac{q}{az-1} \qquad \Rightarrow \quad \boxed{a = \frac{q}{p}} \; \longrightarrow \textcircled{3}.$$

$$q = az - 1$$

$$q + 1 = az$$

$$\boxed{a = \frac{q+1}{z}} \quad (\text{from } \textcircled{2}) \to \textcircled{4}$$

Equating ③ & ④

$$\frac{q+1}{z} = \frac{q}{p}$$

$$(q+1)p = qz \qquad \Rightarrow pq + p = qz$$

$$zq - pq - p = 0.$$

$z = px + qy + \sqrt{p+q^2}$

4) $ax + by + \sqrt{a^4 b^2}$

5) $(x + a^2)(y + b^2) \Rightarrow \frac{p^2}{4} + \frac{q^2}{4} + 1$

6) $\frac{(m^2 - a^2)}{(x-a)^2 + (y-b)^2 + z^2 = 1}$

$(p^2 + q^2 + 1)z^2 = 1$