Problems under methods of Multipliers.

1. Solve $(mz-ny)p+(nx-lz)q = ly-mx$

Sol

The given PDE is a Lagrange's linear equation with.

$P = mz-ny,\ Q = nx-lz,\ R = ly-mx.$

The subsidiary equation are $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$

$$\frac{dx}{mz-ny} = \frac{dy}{nx-lz} = \frac{dz}{ly-mx} \quad \text{--- (1)}$$

Using the multipliers (x,y,z), each of the ratio in (1) is equal to.

$$\frac{x\,dx + y\,dy + z\,dz}{x(mz-ny) + y(nx-lz) + z(ly-mx)} = \frac{x\,dx + y\,dy + z\,dz}{xmz - xny + ynx - lyz + yzl - xzm}$$

$$= \frac{x\,dx + y\,dy + z\,dz}{0}$$

$$x\,dx + y\,dy + z\,dz = 0$$

Integrating, we get.

$$\frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{2} = \frac{c_1}{2}$$

$$x^2 + y^2 + z^2 = c_1$$

$$u = x^2 + y^2 + z^2$$

Using the multipliers (l,m,n), each of the ratio in (1) is equal to

$$\frac{l\,dx + m\,dy + n\,dz}{l(mz-ny) + m(nx-lz) + n(ly-mx)} = \frac{l\,dx + m\,dy + n\,dz}{lmz - nly + mnx - lmz + nly - mnx}$$

$$= \frac{l\,dx + m\,dy + n\,dz}{0}$$

$$\therefore\ l\,dx + m\,dy + n\,dz = 0$$

Integrating, we get,

$$lx + my + nz = c_2$$

$$v = lx + my + nz.$$

The general solution of the given equation is $f(u,v) = 0$

$$f(x^2+y^2+z^2,\ lx+my+nz) = 0$$

2. Solve $(3z-4y)\frac{\partial z}{\partial x} + (4x-2z)\frac{\partial z}{\partial y} = 2y-3x$.

Soln:

The given PDE is a Lagrange's linear equation with.

$P = 3z-4y,\ Q = 4x-2z,\ R = 2y-3x$

The subsidiary equation are $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$.

Using the multipliers (2,3,4) each of the ratios in ① is equal to .

$$\frac{2dx+3dy+4dz}{2(3z-4y)+3(4x-2z)+4(2y-3x)} = \frac{2dx+3dy+4dz}{6z-8y+12x-6z+8y-12x}$$

$$= \frac{2dx+3dy+4dz}{0}$$

Integrating, we get.

$$2x+3y+4z = c_1$$

$$u = 2x+3y+4z.$$

Using the multipliers (x,y,z) each of the multipliers in ① is equal to.

$$\frac{xdx+ydy+zdz}{x(3z-4y)+y(4x-2z)+z(2y-3x)} = \frac{xdx+ydy+zdz}{3xz-4xy+4xy-2yz+2yz-3xz}$$

$$= \frac{xdx+ydy+zdz}{0}$$

$$\therefore xdx+ydy+zdz = 0.$$

Integrating, we get.

$$\frac{x^2}{2}+\frac{y^2}{2}+\frac{z^2}{2} = \frac{c_2}{2}.$$

$$x^2+y^2+z^2 = c_2$$

$$v = x^2+y^2+z^2.$$

$f(u,v)=0 \quad f(2x+3y+4z,\ x^2+y^2+z^2)=0$

3. Solve: $x(y-z)p + y(z-x)q = z(x-y)$.

Sol

The given PDE is a Lagrange's linear equation with

$P = x(y-z)$, $Q = y(z-x)$, $R = z(x-y)$.

The subsidiary equation are $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$.

$$\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)} \quad \text{——} (i).$$

Using the multipliers $(1,1,1)$ each of the ratio in (i) is equal to

$$\frac{dx+dy+dz}{x(y-z)+y(z-x)+z(x-y)} = \frac{dx+dy+dz}{xy-xz+yz-xy+xz-zy}$$

$$= \frac{dx+dy+dz}{0}$$

$$\therefore dx+dy+dz = 0$$

Integrating we get

$$x+y+z = c_1$$

$$u = x+y+z.$$

Using the multipliers $\left(\frac{1}{x}, \frac{1}{y}, \frac{1}{z}\right)$, each of the ratio in (i) is equal to

$$\frac{\frac{1}{x}dx + \frac{1}{y}dy + \frac{1}{z}dz}{\frac{1}{x}x(y-z)+\frac{1}{y}y(z-x)+\frac{1}{z}z(x-y)} = \frac{\frac{1}{x}dx+\frac{1}{y}dy+\frac{1}{z}dz}{y-z+z-x+x-y}$$

$$= \frac{\frac{1}{x}dx+\frac{1}{y}dy+\frac{1}{z}dz}{0}$$

$$\therefore \frac{1}{x}dx+\frac{1}{y}dy+\frac{1}{z}dz = 0$$

Integrating, we get.

$$\log x + \log y + \log z = \log c_2$$

$$\log(xyz) = \log c_2$$

$$xyz = c_2$$

$$v = xyz$$