Unit-III Partial Differential Equations

Solution of Standard Types of first order Partial differential equations

The partial differential of the first order can be written as $f(x,y,z,p,q)=0$, where $p=\frac{\partial z}{\partial x}$ and $q=\frac{\partial z}{\partial y}$, x,y are independent variables and z is a dependent variable.

Standard Type - I

Equations containing p and q only.

(ie) $f(p,q)=0$

1) Solve $\sqrt{p}+\sqrt{q}=1$

$\longrightarrow$

Gn $\sqrt{p}+\sqrt{q}=1 \longrightarrow (1)$

This is of type $f(p,q)=0$

(i) To find the complete integral

Let us assume that

$$z = ax+by+c \longrightarrow (2)$$

$$\frac{\partial z}{\partial x} = a, \quad \frac{\partial z}{\partial y} = b$$

$$p = a, \quad q = b$$

Subs the values of p, q in (1), we get

$$\sqrt{a}+\sqrt{b} = 1$$

$$\sqrt{b} = 1-\sqrt{a}$$

$$b = (1-\sqrt{a})^2$$

Subs the value of b in (2),

$$z = ax+(1-\sqrt{a})^2 y + c \longrightarrow (3)$$

which is the required complete integral.

(ii) To find the Singular Integral.

Diff ③, p.w.r.to c, we get

$0 = 1$ (which is absurd)

There is no singular integral.

(iii) To find the general Integral

put $c = f(a)$ in ③, we get

$$z = ax + (1-\sqrt{a})^2 y + f(a) \longrightarrow ④$$

$$\frac{\partial z}{\partial a} = 0 \longrightarrow ⑤$$

Eliminate a between ④ & ⑤ we get the general integral.

2. Solve. $p+q+pq=0$

$\rightarrow$

Given $p+q+pq=0 \longrightarrow ①$

This is of the form $f(p,q)=0$

To find the Complete integral

Let us assume $z = ax+by+c \longrightarrow ②$

$$\frac{\partial z}{\partial x} = a \qquad \frac{\partial z}{\partial y} = b$$

$$p = a \qquad q = b$$

Subs the values of p, q in ①,

$$a+b+ab = 0$$

$$a+b(1+a) = 0$$

$$b(1+a) = -a$$

$$b = -\frac{a}{1+a}$$

Subs the value of b in ②, we get

$z = ax - \left(\frac{a}{1+a}\right) y + c$ which is the required complete integral.