



Type-II Clairaut's type

$$z = px + qy + f(p, q)$$

1. Solve $z = px + qy + pq$

(1) $\rightarrow$ Given $z = px + qy + pq \rightarrow \textcircled{1}$

The given PDE is Clairaut's type

(i) To find the Complete integral

put $p = a, q = b$ in $\textcircled{1}$, we get

$$z = ax + by + ab \rightarrow \textcircled{2}$$

which is the required Complete integral.

(ii) To find the singular integral

Diff $\textcircled{2}$ p.w.r. to a , we get

$$x + 0 + b = 0$$

$$b = -x$$

Diff $\textcircled{2}$ p.w.r. to b , we get

$$0 + y + a = 0$$

$$a = -y$$

Subst. the values of a, b in $\textcircled{2}$, we get

$$z = -xy - x/y + xy$$

$$z = -xy$$

which is the required singular integral.



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2. Solve $z = px + qy + p^2 + pq + q^2$

Given $z = px + qy + p^2 + pq + q^2 \rightarrow (1)$

The given PDE is Clairaut's type.

(i) To find the complete integral

put $p=a, q=b$ in (1), we get

$$z = ax + by + a^2 + ab + b^2 \rightarrow (2)$$

which is the required Complete integral.

(ii) To find the singular integral

Diff (2), p.w.r. to a , we get

$$x + 2a + b = 0$$

$$2a + b = -x \rightarrow (3)$$

Diff (2), p.w.r. to b , we get

$$y + a + 2b = 0$$

$$a + 2b = -y \rightarrow (4)$$

Solve (3) & (4),

$$(4) \times 2 \Rightarrow 2a + 4b = -2y \rightarrow (5)$$

$$(3) \Rightarrow 2a + b = -x \rightarrow (3)$$

$$(5) - (3) \Rightarrow 3b = -2y + x$$

$$b = \frac{x - 2y}{3}$$

$$(3) \times 2 \Rightarrow 4a + 2b = -2x \rightarrow (6)$$

$$(4) \Rightarrow a + 2b = -y \rightarrow (4)$$

$$(6) - (4) \Rightarrow 3a = -2x + y$$

$$a = \frac{y - 2x}{3}$$

Sub the values of a, b in (2), we get

$$\begin{aligned}
 z &= \left(\frac{y-2x}{3}\right)x + \left(\frac{x-2y}{3}\right)y + \left(\frac{y-2x}{3}\right)^2 + \left(\frac{y-2x}{3}\right)\left(\frac{x-2y}{3}\right) + \left(\frac{x-2y}{3}\right)^2 \\
 &= \frac{xy-2x^2}{3} + \frac{xy-2y^2}{3} + \frac{y^2+4x^2-4xy}{9} + \frac{xy-2y^2-2x^2+4xy}{9} + \frac{x^2+4y^2-4xy}{9} \\
 &= \frac{3xy-6x^2+3xy-6y^2+y^2+4x^2-4xy+xy-2x^2-2y^2+4xy+x^2+4y^2-4xy}{9} \\
 &= \frac{3xy-3x^2-3y^2}{9} \\
 z &= \frac{xy-x^2-y^2}{3}
 \end{aligned}$$

(iii) To find the general integral

Put $b = f(a)$ in ②, we get

$$z = ax + f(a)y + a^2 + af(a) + [f(a)]^2 \quad \text{--- ⑦}$$

$$\frac{\partial z}{\partial a} = 0$$

3. Find singular solution of $z = px + qy + \sqrt{1+p^2+q^2}$

Sol

$$\text{Giv: } z = px + qy + \sqrt{1+p^2+q^2} \quad \text{--- ①}$$

The given PDE is Clairaut's type.

(i) To find the complete integral

put $p=a, q=b$ in ①, we get

$$z = ax + by + \sqrt{1+a^2+b^2} \quad \text{--- ②}$$

which is the required complete integral



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(ii) To find the singular integral
Diff ② partially w.r to a, we get

$$x + \frac{1}{2\sqrt{1+a^2+b^2}}(2a) = 0$$

$$x = \frac{-a}{\sqrt{1+a^2+b^2}} \quad \text{--- ③}$$

Diff ② partially w.r to b, we get

$$y + \frac{1}{2\sqrt{1+a^2+b^2}}(2b) = 0$$

$$y = \frac{-b}{\sqrt{1+a^2+b^2}} \quad \text{--- ④}$$

$$\text{③} = x^2 = \frac{a^2}{1+a^2+b^2} \quad \text{--- ⑤}$$

$$\text{④} = y^2 = \frac{b^2}{1+a^2+b^2} \quad \text{--- ⑥}$$

$$\text{⑤} + \text{⑥} \Rightarrow x^2 + y^2 = \frac{a^2+b^2}{1+a^2+b^2}$$

$$= \frac{1+a^2+b^2-1}{1+a^2+b^2} \quad \text{[Add & sub 1 in numerator]}$$

$$= \frac{1+a^2+b^2}{1+a^2+b^2} - \frac{1}{1+a^2+b^2}$$

$$x^2 + y^2 = 1 - \frac{1}{a^2+b^2+1}$$

$$\frac{1}{a^2+b^2+1} = 1 - x^2 - y^2$$

$$\frac{1}{1-x^2-y^2} = 1+a^2+b^2$$

$$\sqrt{1+a^2+b^2} = \frac{1}{\sqrt{1-x^2-y^2}} \quad \text{--- ⑦}$$



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Sub ⑦ in ③, we get

$$③ \Rightarrow a = \frac{-x}{\sqrt{1-x^2-y^2}}$$

Sub ⑦ in ④, we get

$$④ \Rightarrow b = \frac{-y}{\sqrt{1-x^2-y^2}}$$

Sub the a, b in ②, we get

$$z = \frac{-x^2}{\sqrt{1-x^2-y^2}} + \frac{-y^2}{\sqrt{1-x^2-y^2}} + \frac{1}{\sqrt{1-x^2-y^2}}$$

$$= \frac{1-x^2-y^2}{\sqrt{1-x^2-y^2}}$$

$$z = \sqrt{1-x^2-y^2}$$

$$z^2 = 1-x^2-y^2$$

$$x^2+y^2+z^2=1$$

4. Solve $z = px + qy + p^2q^2$ (H.W).