



## Unit - III Partial Differential Equations

### Solution of Standard Types of first order Partial differential Equations

The partial differential of the first order can be written as  $f(x, y, z, p, q) = 0$ , where  $p = \frac{\partial z}{\partial x}$  and  $q = \frac{\partial z}{\partial y}$ ,  $x, y$  are independent variables and  $z$  is a dependent variable.

#### Standard Type - I

Equations containing  $p$  and  $q$  only.

(i)  $f(p, q) = 0$

1) Solve  $\sqrt{p} + \sqrt{q} = 1$

→

Ans  $\sqrt{p} + \sqrt{q} = 1 \rightarrow \text{①}$

This is of type  $f(p, q) = 0$

(i) To find the complete integral

Let us assume that

$$z = ax + by + c \rightarrow \text{②}$$

$$\frac{\partial z}{\partial x} = a, \quad \frac{\partial z}{\partial y} = b$$

$$p = a, \quad q = b$$

Subs the values of  $p, q$  in ①, we get

$$\sqrt{a} + \sqrt{b} = 1$$

$$\sqrt{b} = 1 - \sqrt{a}$$

$$b = (1 - \sqrt{a})^2$$

Subs the value of  $b$  in ②,

$$z = ax + (1 - \sqrt{a})^2 y + c \rightarrow \text{③}$$

which is the required complete integral.



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(i) To find the Singular Integral.

Diff (3), p.w.r.to c, we get  
 $0 = 1$  (which is absurd)

There is no singular integral.

(ii) To find the general Integral.

Put  $c = f(a)$  in (3), we get

$$Z = ax + (1 - \sqrt{a})^2 y + f(a) \rightarrow (4)$$

$$\frac{\partial Z}{\partial a} = 0 \rightarrow (5)$$

Eliminate  $a$  between (4) & (5) we get the general integral.

2. Solve.  $p + q + pq = 0$

→

Given  $p + q + pq = 0 \rightarrow (1)$

This is of the form  $f(p, q) = 0$

To find the Complete integral

Let us assume  $Z = ax + by + c \rightarrow (2)$

$$\frac{\partial Z}{\partial x} = a \quad \left| \quad \frac{\partial Z}{\partial y} = b \right.$$
$$p = a \quad \left| \quad q = b \right.$$

Subs the values of  $p, q$  in (1),

$$a + b + ab = 0$$

$$a + b(1 + a) = 0$$

$$b(1 + a) = -a$$

$$b = \frac{-a}{1 + a}$$

Subs the value of  $b$  in (2), we get

$$Z = ax - \left( \frac{a}{1 + a} \right) y + c \text{ which is the}$$

required complete integral.