SNS COLLEGE OF TECHNOLOGY

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COIMBATORE-641 035, TAMIL NADU

DEPARTMENT OF MATHEMATICS

UNIT - III

Unit - II / Part - A / 2 Marks				
S.No	**Questions**	**Mark Splitup**	**K – Level**	**CO**
1.	Define analytic function of a complex variable.	2	K2	CO3
2.	Write the necessary condition for $f(z)$ to be analytic	2	K2	CO3
3.	Check whether $w = \bar{z}$ is analytic everywhere.	2	K2	CO3
4.	Test the analyticity of the function $w = \sin z$.	2	K2	CO3
5.	Determine whether the function $2xy + i(x^2 - y^2)$ is analytic or not.	2	K1	CO3
6.	For what values of a, b and c the function $f(z) = x + ay - i(bx + cy)$ is analytic.	2	K3	CO3
7.	Verify the function $u = log\sqrt{x^2 + y^2}$ is harmonic or not?	2	K3	CO3
8.	Show that $u = 2x(1 - y)$ is harmonic.	2	K1	CO3
9.	Find the value of m if $u = 2x^2 - my^2 + 3x$ is harmonic.	2	K1	CO3
10.	Prove that an analytic function whose real part is constant must itself be a constant.	2	K1	CO3
11.	Prove that an analytic function with constant imaginary part is constant.	2	K1	CO3
12.	Construct the analytic function $f(z)$ for which the real part is $e^x cosy$.	2	K1	CO3
13.	Define conformal mapping.	2	K1	CO3
14.	Find the image of the circle $\|z\| = 3$ under the transformation $w = 2z$.	2	K1	CO3
15.	Find the critical points of the transformation $w^2 = (z - \alpha)(z - \beta)$.	2	K1	CO3
16.	Find the invariant points of the bilinear transformation $w = \frac{1+z}{1-z}$.	2	K1	CO3
17.	Find the fixed points of the transformation $w = \frac{6z-9}{z}$.	2	K2	CO3
18.	Define bilinear transformation.	2	K2	CO3

Unit - III / Part - B / 16, 8 Marks				
S.No	**Questions**	**Marks Splitup**	**K – Level**	**CO**
1.	Prove that the real and imaginary parts of an analytic function are harmonic functions.	8	K2	CO3
2.	Show that an analytical function with constant modulus is constant.	8	K2	CO3
3.	If $w = u(x,y) + i\,v(x,y)$ is an analytic function the curves of the family $u(x,y) = a$ and the curves of the family $v(x,y) = b$ cut orthogonally, where a and b are varying constants.	8	K2	CO3

4.	If $f(z)$ is a regular function of z, then prove that $\nabla^2 \lvert f(z)\rvert^2 = 4 \lvert f'(z)\rvert^2$.	8	K2	CO3
5.	If $f(z)$ is an analytic (regular) function of z, then prove that $\nabla^2 \log \lvert f(z)\rvert = 0$.	8	K2	CO3
6.	Determine the analytic function where real part is $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$.	8	K2	CO3
7.	Find an analytic function $u = e^x(x\cos y - y\sin y)$ also find conjugate harmonic function	8	K2	CO3
8.	Show that the function $u = \frac{1}{2}\log(x^2 + y^2)$ is harmonic and determine its conjugate. Also find $f(z)$.	8	K2	CO3
9.	Determine the analytic function whose real part is $\frac{\sin 2x}{\cosh 2y - \cos 2x}$.	8	K2	CO3
10.	Find the analytic function $f(z) = u + iv$ given that $2u + 3v = e^x(\cos y - \sin y)$.	8	K2	CO3
11.	Show that $v = e^{-x}(x\cos y + y\sin y)$ is harmonic function. Hence find the analytic function $f(z)$.	8	K2	CO3
12.	Find the analytic function $f(z) = u + iv$ given that $u - v = e^x(\cos y - \sin y)$.	8	K2	CO3
13	Find the analytic function $f(z) = u + iv$, if $u + v = \frac{\sin 2x}{\cosh 2y - \cos 2x}$	8	K2	CO3
14.	Find the image of the circle $\lvert z - 2i\rvert = 2$ under the transformation $w = \frac{1}{z}$.	8	K3	CO3
15.	Find the image of the circle $\lvert z - 1\rvert = 1$ under the transformation $w = \frac{1}{z}$.	8	K3	CO3
16.	Find the image of the infinite strips $(i)\ \frac{1}{4} < y < \frac{1}{2}$ $(ii)\ 0 < y < \frac{1}{2}$ under the transformation $w = \frac{1}{z}$.	8	K3	CO3
17.	Find the image of the half plane $x > c$, when $c > 0$ under the transformation $w = \frac{1}{z}$. Show the regions graphically.	8	K3	CO3
18.	Find the image of infinite strip $1 < x < 2$ under the transformation $w = \frac{1}{z}$.	8	K3	CO3
19.	Find the bilinear transformation that maps the points $z = 0, -1, i$ into the points $w = i, 0, \infty$ respectively.	8	K3	CO3
20.	Find the bilinear transformation that maps the points $z = -2, 0, 2$ into the points $w = 0, i, -i$ respectively	8	K3	CO3
21.	Find the bilinear mapping which maps points $z = 0, 1, \infty$ into $w = i, 1, -i$ respectively.	8	K3	CO3
22.	Find the bilinear transformation that transforms the points $z = 1, i, -1$ into the points $w = 2, i, -2$ respectively.	8	K3	CO3
23.	Find the bilinear transformation that maps the points $z = 0, 1, \infty$ of the z- plane into the points $w = -5, -1, 3$ of the w- plane. Also find its fixed (Invariant) points.	8	K3	CO3
24.	Find the bilinear transformation that transforms the points $z = 1, i, -1$ into the points $w = 2, i, -2$ respectively. Also	8	K3	CO3

	find its fixed(Invariant) points.			
25.	Find the bilinear transformation which maps the points $1, i, -1$ onto points $0, 1, \infty$, show that the transformation maps the interior of the circle of the z- plane onto the upper half of the w plane.	8	K3	CO3
26.	Find the bilinear transformation that transforms $-1, 0, 1$ of the z- plane onto $-1, -i, 1$ of the w- plane. Show that under this transformation the upper half of the z-plane maps on to the interior of the unit circle $\|w\| = 1$.	8	K3	CO3

UNIT - IV

Unit - IV / Part - A / 2 Marks				
S.No	**Questions**	**Mark Splitup**	**K - Level**	**CO**
1.	State Cauchy's Integral Theorem	2	K1	CO4
2.	Write Cauchy's Integral formula and its derivatives.	2	K1	CO4
3.	Define Taylor's Series	2	K1	CO4
4.	Expand $\frac{1}{z-2}$ at $z = 1$ in Taylor's series.	2	K2	CO4
5.	Define Laurent's Series Expansion.	2	K1	CO4
6.	Define pole and give an example.	2	K1	CO4
7.	Define an isolated singularity and give an example.	2	K1	CO4
8.	Define Essential singularity and give an example.	2	K1	CO4
9.	Define Removable singularity and give an example.	2	K1	CO4
10.	Define Residue.	2	K1	CO4
11.	Discuss the nature of the singularity of the function $\frac{\sin z - z}{z^3}$	2	K2	CO4
12.	Discuss the nature of the singularity of the function $\frac{e^{\frac{1}{z}}}{(z-a)^2}$	2	K2	CO4
13.	Discuss the nature of the singularity of the function $\frac{e^z}{z^2+4}$	2	K2	CO4
14.	Discuss the nature of the singularity of the function $\frac{\cot(\pi z)}{(z-a)^2}$	2	K2	CO4
15.	State Cauchy's Residue Theorem	2	K2	CO4
16.	Find the residue of $\frac{1-e^{2z}}{z^4}$ at $z = 0$	2	K2	CO4
17.	Evaluate $\int_C \frac{dz}{z+4}$, where C is the circle $\|z\| = 2$.	2	K2	CO4
18.	Find the residue of the function $f(z) = \frac{4}{z^3(z-2)}$.	2	K1	CO4