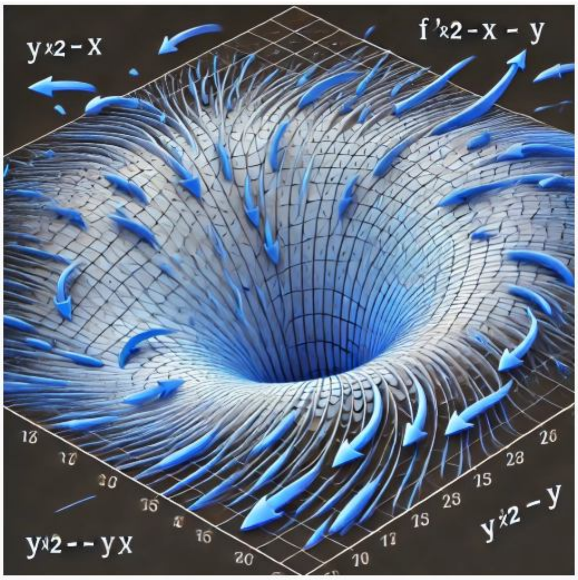
Puzzle: The Mysterious Flow Field

A fluid flows in a two-dimensional plane with a velocity field given by:

$$\mathbf{F}(x, y) = (P, Q) = (y^2 - x, x^2 - y)$$

Questions:

1. Compute the **divergence** of the field, $\nabla \cdot \mathbf{F}$. What does it suggest about the nature of the flow?
2. Compute the **curl** of the field, $\nabla \times \mathbf{F}$. What does it suggest about rotational tendencies in the flow?
3. If a particle starts at $(1, 1)$, determine if it moves in a closed loop by checking if $\mathbf{F}$ is **conservative**.



Here's the **vector field visualization** for the given function $\mathbf{F}(x, y) = (y^2 - x, x^2 - y)$. The arrows represent the flow at different points, showing how the vectors change in direction and magnitude across the plane. Try solving the puzzle and see what insights you can gain about this field! 🚀

1. Compute the Divergence

The **divergence** of a vector field $\mathbf{F} = (P, Q)$ is given by:

$$\nabla \cdot \mathbf{F} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y}$$

For $P = y^2 - x$:

$$\frac{\partial P}{\partial x} = \frac{\partial}{\partial x}(y^2 - x) = -1$$

For $Q = x^2 - y$:

$$\frac{\partial Q}{\partial y} = \frac{\partial}{\partial y}(x^2 - y) = -1$$

Thus, the **divergence** is:

$$\nabla \cdot \mathbf{F} = (-1) + (-1) = -2$$

Ask anything

Interpretation:

Since the divergence is **negative everywhere**, the vector field represents a **compressing or sinking** behavior, meaning fluid particles are moving into regions rather than expanding outward.

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2. Compute the Curl

The **curl** of a 2D vector field $\mathbf{F} = (P, Q)$ is given by:

$$\nabla \times \mathbf{F} = \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}$$

For $Q = x^2 - y$:

$$\frac{\partial Q}{\partial x} = \frac{\partial}{\partial x}(x^2 - y) = 2x$$

For $P = y^2 - x$:

$$\frac{\partial P}{\partial y} = \frac{\partial}{\partial y}(y^2 - x) = 2y$$

Thus, the **curl** is:

$$\nabla \times \mathbf{F} = 2x - 2y$$

Interpretation:

- The curl is **not constant** but depends on the position (x, y).
- If $x > y$, the curl is **positive**, meaning the fluid rotates counterclockwise.
- If $x < y$, the curl is **negative**, meaning the fluid rotates clockwise.
- If $x = y$, the curl is **zero**, indicating no local rotation.

Thus, the field exhibits **rotational behavior that varies across the plane.**

3. Is the Field Conservative?

A vector field is **conservative** if it is the gradient of some scalar potential function, meaning:

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$$

Compute $\frac{\partial P}{\partial y}$:

$$\frac{\partial}{\partial y}(y^2 - x) = 2y$$

Compute $\frac{\partial Q}{\partial x}$:

$$\frac{\partial}{\partial x}(x^2 - y) = 2x$$

Since:

Since:

$$\frac{\partial P}{\partial y} \neq \frac{\partial Q}{\partial x}$$

the field is **not conservative.** This means that a particle moving in the field may not return to its starting position, and **work done by the field along a path depends on the path taken.**

Final Summary

Property	Computation	Conclusion
Divergence $\nabla \cdot \mathbf{F}$	-2	The field represents a compressing or "sinking" effect.
Curl $\nabla \times \mathbf{F}$	$2x - 2y$	The field has varying rotational behavior depending on x and y.
Conservative?	No (since $\frac{\partial P}{\partial y} \neq \frac{\partial Q}{\partial x}$)	The field is not conservative; a particle does not necessarily return to its starting position.

This vector field describes a flow with **compression and varying rotation**, making it an interesting case for fluid mechanics and physics applications! 🚀

Let me know if you have