TEST OF SIGNIFICANCE OF SMALL SAMPLES:

VARIANCE RATIO TEST (or) F-Test for EQUALITY OF VARIANCE

Null hypothesis: $H_0: \sigma_1^2 = \sigma_2^2$

Test statistics: $F = \frac{S_1^2}{S_2^2}$ where $S_1^2 > S_2^2$.

where $S_1^2 = \frac{n_1 s_1^2}{n_1 - 1}$ or $S_1^2 = \frac{\Sigma (x_1 - \bar{x}_1)^2}{n_1 - 1}$ &

$S_2^2 = \frac{n_2 s_2^2}{n_2 - 1}$ or $S_2^2 = \frac{\Sigma (x_2 - \bar{x}_2)^2}{n_2 - 1}$

Degree of Freedom: (ν_1, ν_2)

where $\nu_1 = (n_1 - 1)$, $\nu_2 = (n_2 - 1)$

Note 1:– F Greater than ~~zero~~ one always.

Note 2:– Suppose S_2^2 Greater than S_1^2, then $F = \frac{S_2^2}{S_1^2}$

with degree of freedom, $\nu_1 = n_2 - 1$, $\nu_2 = n_1 - 1$

Applications:

'F test' is used to test if the two samples have come from the same population.

1) Two random sample of 11 and 9 items show that the sample standard deviations of their weights as 0.8 & 0.5 respectively. Assuming that the weight distributions are normal, test the hypothesis that the true variances are equal, against the alternative hypothesis that they are not.

Soln:

Given. $n_1 = 11$, $s_1 = 0.8$

$n_2 = 9$, $s_2 = 0.5$

$$S_1^2 = \frac{n_1 s_1^2}{n_1 - 1} = \frac{11(0.8)^2}{11-1} = 0.704$$

$$S_2^2 = \frac{n_2 s_2^2}{n_2 - 1} = \frac{9(0.5)^2}{9-1} = 0.2812$$

$S_1^2 > S_2^2$.

Step 1 → Formulate H_0 & H_1.

$H_0 : \sigma_1^2 = \sigma_2^2$

$H_1 : \sigma_1^2 \neq \sigma_2^2$

Step 2 → LOS at $\alpha = 5\%$.

Step 3 → Test statistic, $F = \frac{S_1^2}{S_2^2} = \frac{0.704}{0.2812} = 2.5$

Step 4 → Degrees of freedom $(\nu_1, \nu_2) = (n_1 - 1, n_2 - 1)$

$= (10, 8)$

Critical value, $F_{tab} : F_\alpha = 3.35$

Step 5 → Conclusion: $F = 2.5 < 3.35 = F_\alpha$

$\therefore H_0$ is accepted at $\alpha = 5\%$.

) Two random samples gave the following results:

Sample	size	sample mean	sum of squares of deviation from the means.
1	12	14	108
2	10	15	90

Test whether the samples came from the same population.

Soln:

Given:

$n_1 = 12$, $\bar{x}_1 = 14$, $\Sigma(x_1-\bar{x}_1)^2 = 108$

$n_2 = 10$, $\bar{x}_2 = 15$, $\Sigma(x_2-\bar{x}_2)^2 = 90$

$$s_1^2 = \frac{\Sigma(x_1-\bar{x}_1)^2}{n_1-1} = \frac{108}{12-1} = 9.818$$

$$S_2^2 = \frac{\Sigma(x_2-\bar{x}_2)^2}{n_2-1} = \frac{90}{10-1} = 10$$

$$s_1^2 < S_2^2$$

Step 1: Formulate H_0 and H_1:

$H_0: \sigma_1^2 = \sigma_2^2$

$H_1: \sigma_1^2 \neq \sigma_2^2$

Step 2: Los at $\alpha = 5\%$.

Step 3: Test statistics, $F = \frac{S_2^2}{S_1^2} = \frac{10}{9.818}$

$$F = 1.018$$

Step 4 : Degrees of freedom $= (\nu_1, \nu_2)$

$$= (n_2 - 1, n_1 - 1)$$

$$= (9, 11)$$

Critical value, $F_\alpha = 2.90$

Step 5 : Conclusion :

$$F = 1.018 < 2.90 = F_\alpha$$

$\therefore$ H_0 is accepted at 5% LOS.

(ii) 't' Test :

Step 1 : Formulate H_0 & H_1 :

$$H_0 : \mu_1 = \mu_2$$

$$H_1 : \mu_1 \neq \mu_2$$

Step 2 : LOS at 5% = α

Step 3 : Test Statistic, $t = \dfrac{\bar{x}_1 - \bar{x}_2}{S\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$

Here $n_1 = 12, n_2 = 10$; $\bar{x}_1 = 14$, $\bar{x}_2 = 15$

Now $s^2 = \dfrac{\sum(x_1 - \bar{x}_1)^2 + \sum(x_2 - \bar{x}_2)^2}{n_1 + n_2 - 2}$

$$= \frac{108 + 90}{12 + 10 - 2} = 9.9$$

$$S = 3.14$$

$$\therefore t = \frac{14-15}{3.14\sqrt{\frac{1}{12}+\frac{1}{10}}} = -0.744$$

$$|t| = 0.744$$

step 4: Degrees of freedom, $\nu = n_1 + n_2 - 2$
$= 12 + 10 - 2$
$= 20$

$\therefore t_\alpha = 2.086$

step 5: Conclusion, $t = 0.744 < 2.086 = t_\alpha$

$\therefore H_0$ is accepted at 5% Los.

3) Test whether the population variances are identical:

Sample I :	10	11	16	12	10	11	12	16
Sample II :	7	9	3	7	9	3	15	

at 1% Los

Soln: Given: $n_1 = 8$, $n_2 = 7$

x_1	$(x_1-\bar{x}_1)^2$	x_2	$(x_2-\bar{x}_2)^2$
10	5.0625	7	0.3265
11	1.5625	9	2.0409
16	14.0625	3	20.8974
12	0.0625	7	0.3265
10	5.0625	9	2.0409
11	1.5625	3	20.8974
12	0.0625	15	55.1841
16	14.0625		
98	41.5	53	101.4143

$\Sigma(x_2-\bar{x}_2)^2 : 101.71$

$\bar{x}_1 : \frac{\Sigma x_1}{n} = \frac{98}{8}$ $\quad \Sigma(x_1-\bar{x}_1)^2 = 41.5$ $\quad \bar{x}_2 : \frac{\Sigma x_2}{n} : \frac{53}{7} : 7.57$

$$\therefore s_1^2 = \frac{\sum(x_1-\bar{x}_1)^2}{n_1-1} = \frac{41.5}{7} = 5.9286$$

$$s_2^2 = \frac{\sum(x_2-\bar{x}_2)^2}{n_2-1} = \frac{101.7143}{6} = 16.9524$$

$$s_1^2 < s_2^2.$$

Step 1: Formulate H_0 & H_1:

$$H_0 : \sigma_1^2 = \sigma_2^2$$
$$H_1 : \sigma_1^2 \neq \sigma_2^2$$

Step 2: LOS at $\alpha = 1\%$.

Step 3: Test statistic, $F = \frac{s_2^2}{s_1^2}$

$$= \frac{16.9524}{5.9286} := 2.86$$

Step 4: Degrees of freedom: (ν_1, ν_2)

$$= (n_2-1, n_1-1)$$
$$= (6, 7)$$
$$\therefore F_\alpha = 7.19$$

Step 5: Conclusion, $F = 2.86 < 7.19 = F_\alpha$

$\therefore H_0$ is accepted at H_0 at 1% LOS.