UNIT V

DESIGN OF EXPERIMENTS

Experiment : A collection of data or measurements of some features of an object.

Treatments : Various objects of comparison in a comparative experiment are called treatments.

Experimental unit : The smallest division of the experimental material to which we apply the treatments.

Blocks : The whole experimental units are divided into subgroups called blocks.

Experimental error : It is the error occured due to random causes or chance causes or non-assignable factors which are beyond our control.

Basic principles of experimental design

1. Randomisation
2. Replication
3. Local Control

Basic Designs of experiment

1. Completely Randomised Design (C.R.D or one-way classification)
2. Randomised Block Design (R.B.D or Two-way classification)
3. Latin Square Design (L.S.D or Three way classification)

Analysis of Variance (ANOVA)

The analysis of variance is a widely used technique developed by Prof. R.A. Fisher.

Uses : It is used to test whether the means of a number of populations (more than two) are equal.

Assumptions :

1. Each sample taken is a random sample
2. Each one is independent of the other sample.
3. Populations from which samples are taken are normal.
4. Variances of the populations are equal.

Completely Randomised Design (One-way Classification) :

This is a one factor experiment.

Procedure :

Step 1 :

Null hypothesis : H_0 : There is no significant difference between columns and errors.

Alternative hypothesis : H_1 : There is a significant difference between columns and errors.

Step 2 :

* Find N, number of given observation
* Find T, total number of observation
* Find correction factor $C.F = \frac{T^2}{N}$

Step 3 : Find :

* Sum of squares of treatments

$$SST = \frac{\Sigma x_1^2}{} + \Sigma x_2^2 + \Sigma x_3^2 + \cdots - C.F$$

* Sum of Squares of columns

$$SSC = \frac{(\Sigma x_1)^2}{c_1} + \frac{(\Sigma x_2)^2}{c_2} + \frac{(\Sigma x_3)^2}{c_3} + \dots - C.F$$

* Sum of Squares of errors

$$SSE = SST - SSC$$

Step 4 : ANOVA Table :

Source of Variation	Degree of freedom	Sum of Squares	Mean Sum of Squares	Variance ratio	Table value
Between Columns	$c-1$	SSC	$MSC = \frac{SSC}{c-1}$	$\frac{MSC}{MSE}$ (or) $\frac{MSE}{MSC}$	$F_\alpha(c-1, N-c)$ (or) $F_\alpha(N-c, c-1)$
Between Errors	$N-c$	SSE	$MSE = \frac{SSE}{N-c}$		

Step 5 : Decision :

If Since $|F| < F_\alpha$, we accept the hypothesis

$|F| > F_\alpha$, we reject the hypothesis.

Problem :

(1) A random sample is selected from each of these makes of ropes and their breaking strength are measured with the following results. Test whether the breaking strength of the ropes differ significantly.

I	II	III
70	100	60
72	110	65
75	108	57
80	112	84
83	113	87
	120	73
	107	

Solution:

Step 1: Null hypothesis: H_0: There is no significant difference between Column and errors

Alternative hypothesis: H_1: There is a significant difference between Column and errors

Step 2:

$N = 18$

$T = 1576$

$$C.F = T^2/N = \frac{1576^2}{18} = 137987.55$$

Step 3:

$$SST = \Sigma x_1^2 + \Sigma x_2^2 + \Sigma x_3^2 - C.F$$

x_1	x_2	x_3	x_1^2	x_2^2	x_3^2
70	100	60	4900	10000	3600
72	110	65	5184	12100	4225
75	108	57	5625	11664	3249
80	112	84	6400	12544	7056
83	113	87	6889	12769	7569
	120	73		14400	5329
	107			11449	
380	770	426	28998	84926	31028

$$SST = 28998 + 84926 + 31028 - 137987.55$$
$$= 6964.45$$

$$SSC = \frac{(\Sigma x_1)^2}{c_1} + \frac{(\Sigma x_2)^2}{c_2} + \frac{(\Sigma x_3)^2}{c_3} - C.F$$

$$= \frac{(380)^2}{5} + \frac{(770)^2}{7} + \frac{(426)^2}{6} - 137987.55$$

$$SSC = 5838.44$$

$SSE = SST - SSC$

$= 6964.45 - 5838.44$

$SSE = 1126.01$

Step 4 : ANOVA Table :

Source of Variation	Degree of freedom	Sum of Squares	Mean sum of Squares	Variance ratio	Table value
Between Columns	$c-1 = 3-1 = 2$	$SSC = 5838.44$	$MSC = \frac{SSC}{c-1} = 2919.22$	$F = \frac{MSC}{MSE} = 38.88$	$F_\alpha(2,15) = 3.68$
Between Errors	$N-c = 18-3 = 15$	$SSE = 1126.01$	$MSE = \frac{SSE}{N-c} = 75.067$		

Step 5 : Decision :

Since $F > F_\alpha$, we reject the hypothesis

∴ There is a significant difference between Columns and errors