Problem:

(1) An experiment was designed to study the performance of 4 different detergents for cleaning of injectors. The following "cleanliness" readings were obtained with specially designed equipment for 12 tanks of gas distributed over 3 different models of engines.

Detergent	Engine 1	Engine 2	Engine 3	Total
A	45	43	51	139
B	47	46	52	145
C	48	50	55	153
D	42	37	49	128
Total	182	176	207	565

Perform the ANOVA test at 0.01 level of significance whether there are differences in the detergents or in the engines.

Solution: Fix origin = 50. Subtract each element from 50.

Detergent \ Engine	x_1	x_2	x_3	Total	x_1^2	x_2^2	x_3^2
A (y_1)	−5	−7	+1	−11	25	49	1
B (y_2)	−3	−4	2	−5	9	16	4
C (y_3)	−2	0	5	3	4	0	25
D (y_4)	−8	−13	−1	−22	64	169	1
Total	−18	−24	7	−35	102	234	31

Step 1 : Null hypothesis H_0 : There is no significant difference between engines and detergents.

Alternative hypothesis H_1 : There is a significant difference between engines and detergents

Step 2 : ★ $N = 12$

$T = -35$

$C.F = T^2/N = 102.08$

Step 3 : $SST = \Sigma x_1^2 + \Sigma x_2^2 + \Sigma x_3^2 - C.F$

$= 102 + 234 + 31 - 102.08$

$\boxed{SST = 264.92}$

$$SSC = \frac{(\Sigma x_1)^2}{c_1} + \frac{(\Sigma x_2)^2}{c_2} + \frac{(\Sigma x_3)^2}{c_3} - C.F$$

$$= \frac{(-18)^2}{4} + \frac{(-24)^2}{4} + \frac{7^2}{4} - 102.08$$

$\boxed{SSC = 135.17}$

$$SSR = \frac{(\Sigma y_1)^2}{r_1} + \frac{(\Sigma y_2)^2}{r_2} + \frac{(\Sigma y_3)^2}{r_3} + \frac{(\Sigma y_4)^2}{r_4} - C.F$$

$$= \frac{(-11)^2}{3} + \frac{(-5)^2}{3} + \frac{3^2}{3} + \frac{(-22)^2}{3} - 102.08$$

$\boxed{SSR = 110.91}$

$SSE = TSS - SSC - SSR = 264.92 - 135.17 - 110.91$

$\boxed{SSE = 18.84}$

Step 4 : ANOVA table :

Source of Variation	Degree of freedom	Sum of Squares	Mean Sum of Squares	Variance ratio	Table value at 1% level
Between Columns	$c-1 = 3-1 = 2$	$SSC = 135.17$	$MSC = \frac{SSC}{c-1} = 67.585$	$F_c = \frac{MSC}{MSE} = 21.52$	$F_\alpha(2,6) = 10.92$
Between rows	$r-1 = 4-1 = 3$	$SSR = 110.91$	$MSR = \frac{SSR}{r-1} = 36.97$	$F_R = \frac{MSR}{MSE} = 11.77$	$F_\alpha(3,6) = 9.78$
Between errors	$(c-1)(r-1) = 6$	$SSE = 18.84$	$MSE = \frac{SSE}{(c-1)(r-1)} = 3.14$		

Step 5 : **Decision** : Since $F_c > F_\alpha$ and $F_R > F_\alpha$, H_0 is rejected

$\therefore$ There is a significant difference between engines and detergents