PART-B

Unit III,TwoDimensionalRandomVariables

1. Fromthefollowingdistributionof(X,Y)find.(i) $P(X\leq 1)$(ii)$P(Y\leq 3)$(iii)$P(X\leq 1,Y\leq 3)$ (iv)$P(X\leq \frac{1}{Y}\leq 3)(v)P(Y\leq \frac{3}{X}\leq 1)(vi)P(X+Y\leq 4)$.

X \ Y	1	2	3	4	5	6
0	0	0	$\frac{1}{32}$	$\frac{2}{32}$	$\frac{2}{32}$	$\frac{3}{32}$
1	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
2	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$	0	$\frac{2}{64}$

2. Thejointprobabilityfunction (X,Y)isgivenby $P(x,y)=k(2x+3y)$ $x=0,1,2;$ $y=1,2,3$
   (i) Findthemarginaldistributions.
   (ii) Findtheprobabilitydistributionsof (X+Y)
   (iii) Findallconditionalprobabilitydistributions.
3. Thejointp.dfoftherandomvariable (X,Y)is givenby

$$f(x,y)=\begin{cases}\frac{x(1+3y^2)}{4} & 0<x<2,\quad 0<y<1\\ 0 & otherwise\end{cases}$$

Find
   (i) Marginal densityfunctionsofXandY
   (ii) ConditionaldensityofXgivenY
   (iii) $P(\frac{1}{4}<X<2=\frac{1}{Y}\ \frac{}{3})$

4. Thejointp.d.ofthetwodimensionalrandomvariable (X,Y) is given by

$$f(x,y)=\begin{cases}\frac{8xy}{9} & :1\le x\le y\le 2\\ 0 & :otherwise\end{cases}$$

Find

(i) MarginaldensitiesofXandY

(ii) The conditionaldensityfunctions$f(x/y)$ $and f(y/x)$.

5. Ifthejointp.d.ofatwodimensionalrandomvariable (X,Y) is given by $f(x,y)=$

$$\begin{cases}x^2+\frac{xy}{3} & 0<x<1;0<y<2\\ 0: & otherwise\end{cases}$$

Find(i)$P(X>{}^1/_2)$(ii)$P(Y>1)$(iii)$P(Y<X)$

(iii) $P(Y<{}^1/_2 \,/\, X<\frac{1}{2})$ (v)$P(X+Y\ge 1)$

(vi) find theconditionaldensityfunctions.

(vii) Checkwhethertheconditionaldensityfunctions are valid.

6. Thejointp.d.oftherandom variable(X,Y)isgivenby

$f(x,y)=kxye^{-(x^2+y^2)}x>0,y>0$

(i) Findk (ii)ProvethatXand Yare independent.

7. Given

$$f_{XY}(x,y)=\begin{cases}cx(x-y) & ,0<x<2,-x<y<x\\ 0 & otherwise\end{cases}$$

(i) Evaluatec

(ii) Find$f_X(x)$

(iii) $f_{\frac{Y}{X}}(y/x)$

(iv) $f_Y(y)$.

8. Two random variables X and Y have the following joint probability density functions

$$f(x,y)=\begin{cases} 2-x-y : 0\le x\le 1, 0\le y\le 1 \\ 0 \quad : otherwise \end{cases}$$

   (i) Findthemarginal densityfunctionsofXandY
   (ii) Conditionaldensityfunction
   (iii) VarXandVarY
   (iv) CorrelationcoefficientbetweenXand Y.
9. Giventhejointp.d.fofXandY is

$$f(x,y)=\begin{cases} 8xy : 0<x<y<1 \\ 0 \quad : otherwise \end{cases}$$

   Findthemarginalandconditionalp.d.f'sXandY.AreX andY independent?
10. Let(X,Y)bethetwodimensionalrandomvariabledescribed by the joint p.d.f

$$f(x,y)=\begin{cases} 8xy : 0\le x\le 1, 0\le y\le x \\ 0 \quad : otherwise \end{cases}$$

   Find theCov(X,Y).
11. The joint p.d.f of the random variable (X,Y) is $f(x,y)=3(x+y):0\le x\le 1, 0\le y\le 1, x+y\le 1.$ FindCov(X,Y).
12. If X and Y are uncorrelated random variables with variances 16 and 9, find the correlation co-efficient between x+y and x-y.
13. Marks obtained by 10 students in Mathematics(x) and statistics(y) are given below

x:	60	34	40	50	45	40	22	43	42	64
y:	75	32	33	40	45	33	12	30	34	51

   Findthetwo regression lines.Also find ywhen x=55.
14. In a correlation analysis the equations of the two regressionlinesare$3x + 12y=9; and\ 3y + 9x=46$.

Find(i)Thevalueofthecorrelationcoefficient (ii) Mean value of X and Y.

15. Find the correlation coefficient and theequationof the regression lines for the following values of X and Y.

X	1	2	3	4	5
Y	2	5	3	8	7

16. Findthemostlikelypricein CityAcorrespondingto the price of Rs.70 at City B from the following:

	CityB	CityA
AveragePrice	65	67
S.D.ofPrice	2.5	3.5

Correlationcoefficientis0.8.

17. The joint p.d.f of the random variable (X,Y) is givenas

$$f(x,y) = \begin{cases} e^{-(x+y)} & ;x>0,y>0 \\ 0 & ;otherwise \end{cases}$$

Findthedistributionof $\frac{1}{2}(X-Y)$.

18. TheindependentrandomvariablesXandYfollow exponentialdistributionwithparameter$\lambda=1$. Findthe p.d.fof$U=X-Y$.

19. LetXandYarenormallydistributedindependent randomvariables with mean 0 and variance σ^2.Find the p.d.f'sof$R=\sqrt{x^2+y^2} and \theta=tan^{-1}\left(\frac{y}{x}\right)$.

20. The joint p.d.f of a two dimensional random variable (X,Y)isgivenby$f(x,y) = x+y, 0 \le x,y \le 1$.Find the p.d.f of U=XY.

21. IfXandYareindependentrandomvariables,with p.d.f$f(x)=e^{-x}, x\ge 0$:$f(y)=e^{-y}, y\ge 0$.Show that$U=\frac{X}{X+Y}$ $and V=X+Y$areindependent.

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