TEST OF SIGNIFICANCE OF SMALL SAMPLES:

STUDENT'S t- TEST:

TEST FOR SINGLE MEAN:

Null hypothesis: $H_0: \mu = \mu_0$.

Test Statistic, $t = \dfrac{\bar{x} - \mu}{S/\sqrt{n-1}}$ if SD is given.

$t = \dfrac{\bar{x} - \mu}{S/\sqrt{n}}$ if SD is not given.

[To find s:

$$s^2 = \frac{\Sigma (x - \bar{x})^2}{n-1}$$]

Degrees of Freedom: $\nu = n-1$

NOTE: Confidence limit: $\bar{x} \pm t_{\alpha} \dfrac{S}{\sqrt{n-1}}$

1) A random sample of 10 boys had the following IQ's. 70, 120, 110, 101, 88, 83, 95, 98, 107, 100. Do these data support the assumption of a population mean IQ's of 100? Find a reservable range in which most of the mean IQ's value of sample 10 boys.

Soln: given: $n = 10$, $\mu = 100$

$$\bar{x} = \frac{70+120+110+101+88+83+95+98+107+100}{10}$$

$$= 97.2.$$

To find s : $s^2 = \frac{\sum (x-\bar{x})^2}{n-1}$

x :	70	120	110	101	88	83	95	98	107	100
$x-\bar{x}$:	-27.2	22.8	12.8	3.8	-9.2	-14.2	-2.2	0.8	9.8	2.8
$(x-\bar{x})^2$:	739.84	519.84	163.84	14.44	84.64	201.64	4.84	0.64	96.04	7.84

$\sum (x-\bar{x})^2 = 1833.6$

$\therefore s^2 = \frac{\sum (x-\bar{x})^2}{n-1} = \frac{1833.6}{10-1}$

$= 203.73$

$\Rightarrow s = 14.27$.

Step 1 : Formulating H_0 and H_1 :

$H_0 : \mu = 100$

$H_1 : \mu \neq 100$ (Two tailed test)

Step 2 : Los. at $\alpha = 5\% = 0.05$

Step 3 : Test statistic, $t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$

$= \frac{97.2 - 100}{14.27/\sqrt{10}}$

$= -0.62$

$|t| = 0.62$.

step 4: t_{tab} for degree of freedom, $\nu = n-1$

$\nu = 10-1 = 9$

(ie) $t_{tab} : 2.262$ (t_α)

step 5: Conclusion: $t = 0.62 < 2.262 = t_\alpha$

$\therefore$ Ho is accepted at 5% LOS.

(ie) the population mean IQ's is 100.

Confidence limit:

$$\mu = \bar{x} \pm t_\alpha \frac{s}{\sqrt{n-1}}$$

$$= 97.2 \pm 2.262 \times \frac{14.27}{\sqrt{10-1}}$$

$$= 97.2 \pm 10.759$$

$$= 107.95, 86.45$$

3) the weights of 10 peoples of a locality are found to be 70, 67, 62, 68, 61, 68, 70, 64, 64, 66 kg it is resonable to believe that the average weights of people locality greater than 64 kg. test at 5% LOS.

Soln: Given: $n = 10$, $\mu = 64$

$$\bar{x} = \frac{70+67+62+68+61+68+70+64+64+66}{10}$$

$$\bar{x} = 66$$

To find s:

$$s^2 = \frac{\sum(x-\bar{x})^2}{n-1}$$

x:	70	67	62	68	61	68	70	64	64	66
$x-\bar{x}$:	4	1	-4	2	-5	2	4	-2	-2	0
$(x-\bar{x})^2$:	16	1	16	4	25	4	16	4	4	0

$$\sum(x-\bar{x})^2 = 90$$

$$\therefore\ s^2 = \frac{\sum(x-\bar{x})^2}{n-1} = \frac{90}{10-1} = 10$$

$$s = 3.16$$

step 1 : Formulating H_0 and H_1 :

$H_0: \mu = 64$

$H_1: \mu > 64$ (one tailed test - right)

step 2 : Los at $\alpha = 5\%$.

step 3 : Test statistic, $t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$

$$= \frac{66-64}{3.16/\sqrt{10}}$$

$$= 2.02$$

step 4 : t_{tab} for degree of freedom, $\nu = n-1$

$= 10-1$

$= 9$

(ii) t_{tab} : $t_\alpha = 1.833$ (at two tailed at 10%.)

Step 5 : Conclusion: $t = 2.02 > \cancel{0.9165}\ 1.833 = t_\alpha$

$\therefore H_0$ is rejected at 5% LOS

i.e.) the avg. weight of people locality is greater than 64.kg .

TEST FOR DIFFERENCE OF MEAN:

Null hypothesis : $H_0 : \mu_1 = \mu_2$

Test statistics, $t = \dfrac{\bar{x}_1 - \bar{x}_2}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$

where $s^2 = \dfrac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}$ (or) $s^2 = \dfrac{\Sigma(x_1 - \bar{x}_1)^2 + \Sigma(x_2 - \bar{x}_2)^2}{n_1 + n_2 - 2}$

Degree of freedom ; $\nu = n_1 + n_2 - 2$.

1) In a test examination given to two groups of students. the marks obtained were as follows:

Group I : 18 20 36 50 49 36 34 49 41

Group II : 29 28 26 35 30 44 46

Examine whether the significance of difference between the average marks secured by the students of the above two groups.

soln: given: Group I: $n_1 = 9$

Group II: $n_2 = 7$.

Now $\bar{x}_1 = \frac{18+20+36+50+49+36+34+49+41}{9} = 37$.

$\bar{x}_2 = \frac{29+28+26+35+30+44+46}{7} = 34$.

x_1	$x_1-\bar{x}_1$	$(x_1-\bar{x}_1)^2$	x_2	$(x_2-\bar{x}_2)$	$(x_2-\bar{x}_2)^2$
18	-19	361	29	-5	25
20	-17	289	28	-6	36
36	-1	1	26	-8	64
50	13	169	35	1	1
49	12	144	30	-4	16
36	-1	1	44	10	100
34	-3	9	46	12	144
49	12	144			
41	4	16			

$\Sigma(x_1-\bar{x}_1)^2 = 1134$ $\qquad\qquad$ $\Sigma(x_2-\bar{x}_2)^2 = 386$.

Now $S^2 = \frac{\Sigma(x_1-\bar{x}_1)^2 + \Sigma(x_2-\bar{x}_2)^2}{n_1+n_2-2}$

$= \frac{1134+386}{9+7-2} = 108.57$

$\Rightarrow S = 10.42$.

Step 1: Formulating H_0 and H_1

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$ (two tailed test)

Step 2: LOS at $\alpha = 5\%$.

Step 3: Test statistic, $t = \dfrac{\bar{x}_1 - \bar{x}_2}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$

$$= \frac{37 - 34}{10.42\sqrt{\frac{1}{9} + \frac{1}{7}}}$$

$$= 0.5713$$

Step 4: t_{tab} for degrees of freedom, $\nu = n_1 + n_2 - 2$

$= 9 + 7 - 2$

$\nu = 14$

(ii) $t_{tab} = (t_\alpha) = 2.145$

Step 5: Conclusion: $t = 0.5713 < 2.145 = t_\alpha$.

$\therefore$ H_0 is accepted at 5% LOS.

$\therefore$ there is no significant difference in the avg. marks of the two groups of students.

2) A samples of two types of Electric bulbs were tested for length of life and the following data were obtained.

Samples	size	Mean	SD
I	8	1134	35
II	7	1024	40

Test at 5%.

Soln: Given:

Sample I : $n_1 = 8$, $\bar{x}_1 = 1134$, $S_1 = 35$

Sample II : $n_2 = 7$, $\bar{x}_2 = 1024$, $S_2 = 40$.

Step 1: Formulating H_0 and H_1.

$$H_0 : \mu_1 = \mu_2$$

$$H_1 : \mu_1 \neq \mu_2 \text{ (two tailed test)}$$

Step 2: LOS at $\alpha = 5\%$.

Step 3: Test statistic, $t = \dfrac{\bar{x}_1 - \bar{x}_2}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$

Now $S^2 = \dfrac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}$

$= \dfrac{8(35)^2 + 7(40)^2}{8 + 7 - 2}$

$= 1615.38$

$S = 40.19$

$$\therefore \quad t = \frac{1134 - 1024}{40.19\sqrt{\frac{1}{8} + \frac{1}{7}}}$$

$$= \frac{110}{20.8} = 5.288$$

step 4: t_{tab} for degrees of freedom, $v = n_1 + n_2 - 2$

$$= 8 + 7 - 2$$

$$= 13$$

(ii) t_{tab}: $t_\alpha = 2.160$.

step 5: Conclusion: $t = 5.288 > 2.160 = t_\alpha$.

$\therefore$ H_0 is rejected at 5%.