LATIN SQUARE :

) An agriculturist wants to test the effects of four different fertilizers A, B, C and D on the yield of paddy. In order to eliminate sources of error due to variability in soil-fertility he used the fertilizers in a Latin square arrangement as given below where the numbers indicate yields in quintals per unit area. Perform an analysis of variance to decide whether there is a difference between the fertilizers at 5% level of significance.

A 18	D 21	C 23	B 11
D 22	A 20	B 10	C 19
B 15	C 21	D 25	A 17
C 22	B 12	A 15	D 24

Soln: Let origin = $x_{ij} - 18$ avs(min, max)

	x_1	x_2	x_3	x_4	total	x_1^2	x_2^2	x_3^2	x_4^2
y_1	0	3	5	-7	1	0	9	25	49
y_2	4	+2	-8	1	-1	16	4	64	1
y_3	-3	3	7	-1	6	9	9	49	1
y_4	4	-6	-3	6	1	16	36	9	36
total	5	2	1	-1	7	41	58	147	87
	Σx_1	Σx_2	Σx_3	Σx_4		Σx_1^2	Σx_2^2	Σx_3^2	Σx_4^2

Step 1: Formulate H_0 & H_1:

H_0: There is no difference between the fertilizers

H_1: There is difference between the fertilizers.

step 2 : To find T & N :

$T = \Sigma x_1 + \Sigma x_2 + \Sigma x_3 + \Sigma x_4$.

$= 5 + 2 + 1 - 1 = 7$

$N = n_1 + n_2 + n_3 + n_4$

$= 4 + 4 + 4 + 4 = 16$

step 3 : To find correction factor, C.F.

$C.F = \frac{T^2}{N} = 49 \frac{49}{16} = 3.0625$

step 4 : To find TSS :

$TSS = \Sigma x_1^2 + \Sigma x_2^2 + \Sigma x_3^2 + \Sigma x_4^2 - CF$

$= 41 + 58 + 147 + 84 - 3.0625$

$= 333 - 3.0625 = 329.94$

step 5 : To find SSC, SSR, & SST

$SSC = \frac{(\Sigma x_1)^2}{n_1} + \frac{(\Sigma x_2)^2}{n_2} + \frac{(\Sigma x_3)^2}{n_3} + \frac{(\Sigma x_4)^2}{n_4} - C.F$

$= \frac{5^2}{4} + \frac{2^2}{4} + \frac{1^2}{4} + \frac{-1^2}{4} - 3.0625$

$= 4.6875$

$SSR = \frac{(\Sigma y_1)^2}{n_1'} + \frac{(\Sigma y_2)^2}{n_2'} + \frac{(\Sigma y_3)^2}{n_3'} + \frac{(\Sigma y_4)^2}{n_4'} - C.F$

$= \frac{1^2}{4} + \frac{-1^2}{4} + \frac{6^2}{4} + \frac{1^2}{4} - 3.0625$

$= 6.6875$

To find SST :

A	0	2	-3	-1	$-2 : \Sigma z_1$
B	-3	-6	-8	-7	$-24 : \Sigma z_2$
C	4	3	5	1	$13 : \Sigma z_3$
D	4	3	7	6	$20 \quad \Sigma z_4$

$$SST = \frac{(\Sigma z_1)^2}{4} + \frac{(\Sigma z_2)^2}{4} + \frac{(\Sigma z_3)^2}{4} + \frac{(\Sigma z_4)^2}{4} - C.F$$

$$= \frac{-2^2}{4} + \frac{-24^2}{4} + \frac{13^2}{4} + \frac{20^2}{4} - C.F$$

$$= 284.25 - 3.0625 = 284.1875$$

Step 6: To find SSE

$$SSE = TSS - SSC - SSR - SST$$

$$= 329.94 - 4.6875 - 6.6875 - 284.1875$$

$$= 34.375$$

Step 7: Anova table.

Source of variations	Sum of Squares	Degrees of Freedom	Mean Sum of Squares	F-Ratio
Column	SSC: 4.6875	c-1 = 3	MSC = 4.6875/3 = 1.5625	Fc = 5.7291/1.5625 = 3.6666, Fα(6,3) = 9.94
Row	SSR: 6.6875	r-1 = 3	MSR = 6.6875/3 = 2.2291	FR = 2.2291/5.7291 = 2.5701, Fα(6,3) = 9.94
Treatment	SST: 284.1875	T-1 = 3	MST = 284.1875/3 = 94.7291	FT = 94.7291/5.7291 = 16.5347, Fα(3,6) = 4.76
Error.	SSE: 34.375	(r-1)(r-2) 3 × 3 = 6	MSE = 34.375/6 = 5.7291	

Step 8: Conclusion:

$F_c = 3.6666 < 9.94 = F_\alpha$, Ho is accepted

$F_R = 2.5701 < 9.94 = F_\alpha$, Ho is accepted

$F_T = 16.5347 > 4.76 = F_\alpha$, Ho is rejected

∴ There is difference between the fertilizers.

2) Analyse the variance in the Latin square of yields (in quintals) of wheat where P, Q, R, S represent the different manures used.

S 222	P 221	R 223	Q 222
Q 224	R 223	P 222	S 225
P 220	Q 219	S 220	R 221
R 222	S 223	Q 221	P 222

Test whether the different manures used have given significantly different yields.

Soln: $F_C = 1.34$; $F_R = 12.31$, $F_T = 2.12$ & $F_\alpha = 4.76$.