NUMERICAL INTEGRATION BY TRAPEZOIDAL

TRAPEZOIDAL RULE :

$$\int_{x_0}^{x_n} y\,dx = \frac{h}{2}\left[(y_0+y_n)+2(y_1+y_2+\cdots+y_{n-1})\right]$$

$$= \frac{h}{2}[A+2B]$$

where A = Sum of the first & last ordinates

B = Sum of the remaining ordinates.

(1) Using trapezoidal rule, evaluate $\int_{-1}^{1} \frac{dx}{1+x^2}$ taking 8 intervals.

Soln: Gn. $y(x) = \frac{1}{1+x^2}$

Here $h = \frac{b-a}{n}$ where $a=-1$, $b=1$, and $n=8$

$\Rightarrow h = \frac{2}{8} = 0.25$

x:	-1	-0.75	-0.5	-0.25	0	0.25	0.5	0.75	1
y:	0.5	0.64	0.8	0.9412	1	0.9412	0.8	0.64	0.5

Trapezoidal rule,

$$\int_{-1}^{1} \frac{1}{1+x^2}\,dx = \frac{h}{2}\left[(y_0+y_n)+2(y_1+y_2+\dots+y_{n-1})\right]$$

$= \frac{h}{2}$ [sum of the first and last ordinate + 2×sum of the remaining ordinates]

$$= \frac{0.25}{2}\left[(0.5+0.5)+2(0.64+0.8+0.9412+0.9412+0.8+0.64)\right]$$

$$= \frac{0.25}{2} \times 12.5248$$

$$= 1.5656$$

(2) Dividing the range into 10 equal parts, find the value of $\int_0^{\pi/2} \sin x\,dx$ by (i) Trapezoidal rule

Soln:

x :	0	$\pi/20$	$2\pi/20$	$3\pi/20$	$4\pi/20$	$5\pi/20$
$y=\sin x$:	0	0.1564	0.3090	0.4540	0.5878	0.7071

x :	$6\pi/20$	$7\pi/20$	$8\pi/20$	$9\pi/20$	$10\pi/20$
$y:\sin x$:	0.8090	0.8910	0.9511	0.9877	1

By Trapezoidal rule;

$$\int_0^{\pi/2} \sin x\,dx = \frac{h}{2}\left[(y_0+y_{11})+2(y_1+y_2+\cdots+y_{10})\right]$$

Here $\boxed{h=\frac{\frac{\pi}{2}-0}{10}=\frac{\pi}{20}}$

$$=\frac{h}{2}\left[(0+1)+2(0.1564+0.3090+0.4540+0.5878+0.7071+0.8090+0.8910+0.9511+0.9877)\right]$$

$$=\frac{\pi}{20}\cdot\frac{1}{2}\left[12.7062\right]$$

$$=0.9980$$