EULER METHOD:

$$y_1 = y_0 + h\,f(x_0, y_0) \quad \text{for the interval } (x_0, y_0)$$

$$y_2 = y_1 + h\,f(x_1, y_1) \quad\quad \text{"} \quad\quad \text{"} \quad (x_1, y_1)$$

$$\cdots \cdots \cdots$$

$$y_{n+1} = y_n + h\,f(x_n, y_n) \quad\quad \text{"} \quad\quad \text{"} \quad (x_n, y_n)$$

where $n = 0, 1, 2, \ldots$

This formula is called Euler's algorithm.

① Using Euler's method find $y(0.2)$ and $y(0.4)$ from

$\dfrac{dy}{dx} = x + y$, $\;y(0) = 1$ with $h = 0.2$.

<u>Soln</u>: $\dfrac{dy}{dx} = f(x, y) = x + y$

Here $x_0 = 0$, $y_0 = 1$, $h = 0.2$. $x_1 = 0.2$, $y_1 = ?$

$\qquad\qquad\qquad\qquad x_2 = 0.4 \quad y_2 = ?$

$$y_1 = y_0 + h f(x_0, y_0)$$
$$= 1 + (0.2) [x_0 + y_0]$$
$$= 1 + (0.2) [0 + 1]$$

$$y(0.2) = 1.2$$

$$y_2 = y_1 + h f(x_1, y_1)$$
$$= 1.2 + (0.2) [x_1 + y_1]$$
$$= 1.2 + 0.2 [0.2 + 1.2]$$

$$y(0.4) = 1.48$$

② using Euler's method solve $y' = x + y + xy$, $y(0) = 1$ Compute y at $x = 0.1$ by taking $h = 0.05$.

Soln:

Gm: $f(x,y) = x + y + xy$

$$x_0 = 0; \quad y_0 = 1, \quad h = 0.05.$$

$$y_1 = y_0 + h f(x_0, y_0)$$
$$= 1 + (0.05) [x_0 + y_0 + x_0 y_0]$$
$$= 1 + (0.05) [0 + 1 + 0]$$

$$y(0.05) = 1.05$$

$$y_2 = y_1 + hf(x_1, y_1)$$

$$= 1.05 + (0.05)\left[x_1 + y_1 + x_1 y_1\right]$$

$$= 1.05 + (0.05)\left[0.05 + 1.05 + 0.05 \times 1.05\right]$$

$$y(0.1) = 1.0527$$

③ Using Euler's method find the soln. of the initial value problem $\dfrac{dy}{dx} = \log(x+y)$, $y(0) = 2$ at $x = 0.2$ by assuming

$h = 0.2$.

<u>Soln:</u> $y(0.2) = 2.0602$.