



DEPARTMENT OF MATHEMATICS

UNIT-IV INTERPOLATION, NUMERICAL DIFFERENTIATION & INTEGRATION

LAGRANGIAN INTERPOLATION

Let $y = f(x)$ be a function which takes the values $y_0, y_1, y_2, \dots, y_n$ corresponding to $x_0, x_1, x_2, \dots, x_n$.

Then Lagrangian interpolation form is

$$y = f(x)$$

$$= \frac{(x-x_1)(x-x_2)\dots(x-x_n)}{(x_0-x_1)(x_0-x_2)\dots(x_0-x_n)} y_0 +$$

$$\frac{(x-x_0)(x-x_2)\dots(x-x_n)}{(x_1-x_0)(x_1-x_2)\dots(x_1-x_n)} y_1 +$$

$$\dots +$$

$$\frac{(x-x_0)(x-x_1)\dots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\dots(x_n-x_{n-1})} y_n$$

Find the polynomial $f(x)$ by using Lagrange's formula and hence find $f(3)$ for



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore- 35



DEPARTMENT OF MATHEMATICS

UNIT-IV INTERPOLATION, NUMERICAL DIFFERENTIATION & INTEGRATION

$$\begin{array}{rcccccc}
 x : & 0 & & 1 & & 2 & 3 & 5 \\
 f(x) : & 2 & & 3 & & 12 & & 147
 \end{array}$$

Soln:

$$\begin{aligned}
 x_0 &= 0 ; y_0 = 2 \\
 x_1 &= 1 ; y_1 = 3 \\
 x_2 &= 2 ; y_2 = 12 \\
 x_3 &= 5 ; y_3 = 147
 \end{aligned}$$

By Lagrange's Interpolation formula, we have

$$\begin{aligned}
 y = f(x) = & \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \\
 & \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1 + \\
 & \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \\
 & \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3
 \end{aligned}$$



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore- 35



DEPARTMENT OF MATHEMATICS

UNIT-IV INTERPOLATION, NUMERICAL DIFFERENTIATION & INTEGRATION

$$\begin{aligned}
 &= \frac{(x-1)(x-2)(x-5)}{(0-1)(0-2)(0-5)} (2) + \frac{(x-0)(x-2)(x-5)}{(1-0)(1-2)(1-5)} (3) \\
 &\quad + \frac{(x-0)(x-1)(x-5)}{(2-0)(2-1)(2-5)} (12) + \frac{(x-0)(x-1)(x-2)}{(5-0)(5-1)(5-2)} (147) \\
 &= \frac{(x-1)(x-2)(x-5)}{-10} (2) + \frac{x(x-2)(x-5)}{4} (3) \\
 &\quad + \frac{x(x-1)(x-5)}{60} (12) + \frac{x(x-1)(x-2)}{60} (147) \\
 &= x^3 + x^2 - x + \frac{6}{10} \\
 y = f(3) &= \frac{(3-1)(3-2)(3-5)}{-10} (2) + \frac{3(3-2)(3-5)}{4} (3) + \\
 &\quad \frac{3(3-1)(3-5)}{-6} (12) + \frac{3(3-1)(3-2)}{60} (147) \\
 &= \frac{8}{10} - \frac{18}{4} + 24 + \frac{147}{60} = 35
 \end{aligned}$$



DEPARTMENT OF MATHEMATICS

UNIT-IV INTERPOLATION, NUMERICAL DIFFERENTIATION & INTEGRATION

Find the missing term in the following table using Lagrange's interpolation.

x	0	1	2	3	4	<u>Soln:</u> 31
y	1	3	9	-	81	

INVERSE INTERPOLATION:

The process of finding a value of x for the corresponding value of y is called inverse interpolation.

Inverse interpolation formula is

$$x = \frac{(y-y_1)(y-y_2)\dots(y-y_n)}{(y_0-y_1)(y_0-y_2)\dots(y_0-y_n)} x_0 +$$

$$\frac{(y-y_0)(y-y_2)\dots(y-y_n)}{(y_1-y_0)(y_1-y_2)\dots(y_1-y_n)} x_1 +$$

$$\frac{(y-y_0)(y-y_1)\dots(y-y_{n-1})}{(y_n-y_0)(y_n-y_1)\dots(y_n-y_{n-1})} x_n$$



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore- 35



DEPARTMENT OF MATHEMATICS

UNIT-IV INTERPOLATION, NUMERICAL DIFFERENTIATION & INTEGRATION

Find the age corresponding to the annuity value 13.6 given the table

Age (x):	30	35	40	45	50
Annuity value (y):	15.9	14.9	14.1	13.3	12.5

Soln:

$$\begin{aligned}
 x = & \frac{(y - y_1)(y - y_2)(y - y_3)(y - y_4)}{(y_0 - y_1)(y_0 - y_2)(y_0 - y_3)(y_0 - y_4)} x_0 + \\
 & \frac{(y - y_0)(y - y_2)(y - y_3)(y - y_4)}{(y_1 - y_0)(y_1 - y_2)(y_1 - y_3)(y_1 - y_4)} x_1 + \\
 & \frac{(y - y_0)(y - y_1)(y - y_3)(y - y_4)}{(y_2 - y_0)(y_2 - y_1)(y_2 - y_3)(y_2 - y_4)} x_2 + \\
 & \frac{(y - y_0)(y - y_1)(y - y_2)(y - y_4)}{(y_3 - y_0)(y_3 - y_1)(y_3 - y_2)(y_3 - y_4)} x_3 + \\
 & \frac{(y - y_0)(y - y_1)(y - y_2)(y - y_3)}{(y_4 - y_0)(y_4 - y_1)(y_4 - y_2)(y_4 - y_3)} x_4
 \end{aligned}$$



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore- 35



DEPARTMENT OF MATHEMATICS

UNIT-IV INTERPOLATION, NUMERICAL DIFFERENTIATION & INTEGRATION

$$\begin{aligned}
 &= \frac{(13.6 - 14.9)(13.6 - 14.1)(13.6 - 13.3)(13.6 - 12.5)}{(15.9 - 14.9)(15.9 - 14.1)(15.9 - 13.3)(15.9 - 12.5)} \times 30 + \\
 &\quad \frac{(13.6 - 15.9)(13.6 - 14.1)(13.6 - 13.3)(13.6 - 12.5)}{(14.9 - 15.9)(14.9 - 14.1)(14.9 - 13.3)(14.9 - 12.5)} \times 35 + \\
 &\quad \frac{(13.6 - 15.9)(13.6 - 14.9)(13.6 - 13.3)(13.6 - 12.5)}{(14.1 - 15.9)(14.1 - 14.9)(14.1 - 13.3)(14.1 - 12.5)} \times 40 + \\
 &\quad \frac{(13.6 - 15.9)(13.6 - 14.9)(13.6 - 14.1)(13.6 - 12.5)}{(13.3 - 15.9)(13.3 - 14.9)(13.3 - 14.1)(13.3 - 12.5)} \times 45 + \\
 &\quad \frac{(13.6 - 15.9)(13.6 - 14.9)(13.6 - 14.1)(13.6 - 13.3)}{(12.5 - 15.9)(12.5 - 14.9)(12.5 - 14.1)(12.5 - 13.3)} \times 50 \\
 &= 43
 \end{aligned}$$