NEWTON'S FORWARD AND BACKWARD DIFFERENCE FORMULA

(EQUAL INTERVALS)

Let the function $y = f(x)$ takes the values $y_0, y_1, \ldots, y_n$ at the points $x_0, x_1, \ldots x_n$ where $x_i = x_0 + ih$.

Then Newton's Forward interpolation polynomial is given by

$$y(x) = P_n(x) = f(x)$$

$$= y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \cdots + \frac{u(u-1)(u-2)\cdots(u-(n-1))}{n!}\Delta^n y_0$$

where $u = \frac{x - x_0}{h}$; h is the difference between two intervals.

Then Newton's Backward interpolation polynomial is given by

$$y(x) = P_n(x) = f(x)$$

$$= y_n + \frac{u}{1!}\nabla y_n + \frac{u(u+1)}{2!}\nabla^2 y_n + \frac{u(u+1)(u+2)}{3!}\nabla^3 y_n$$

$$+ \cdots + \frac{u(u+1)(u+2)\cdots(u+(n-1))}{n!}\nabla^n y_n$$

where $u = \frac{x - x_n}{h}$

Note:

Forward

First order:

$$\Delta y_0 = y_1 - y_0$$

$$\Delta y_1 = y_2 - y_1$$

Second order:

$$\Delta^2 y_0 = \Delta y_1 - \Delta y_0$$

Third order:

$$\Delta^3 y_0 = \Delta^2 y_1 - \Delta^2 y_0$$

Backward.

First order:

$$\nabla y_n = y_n - y_{n-1}$$

$$\nabla y_{n-1} = y_{n-1} - y_{n-2}$$

Second order:

$$\nabla^2 y_n = \nabla y_n - \nabla y_{n-1}$$

Third order:

$$\nabla^3 y_n = \nabla^2 y_n - \nabla^2 y_{n-1}$$

Using Newton's Forward Interpolation & Backward interpolation formula, find the polynomial $f(x)$ satisfying the following data. Hence evaluate y at $x=5$.

x	4	6	8	10
y	1	3	8	10.

Soln:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
4	1			
		$(3-1)$ 2		
			$(5-2)$	
6	3	$(8-3)$	3	
				-6
		5		
8	8		$(2-5)$	
		$(10-8)$	-3	
		2		
10	10			

Forward Interpolation:

Here $x_0 = 4$; $y_0 = 1$; $h = 2$.

$u = \frac{x-4}{2}$

$$y(x) = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0$$

$$= 1 + \left(\frac{x-4}{2}\right)(2) + \left(\frac{x-4}{2}\right)\left(\frac{x-4}{2}-1\right)\frac{(3)}{2!} + \left(\frac{x-4}{2}\right)\left(\frac{x-4}{2}-1\right)\left(\frac{x-4}{2}-2\right)\frac{(-6)}{3!}$$

$$= 1 + x-4 + \frac{(x-4)(x-6)}{4} \times \frac{3}{2} + \frac{(x-4)(x-6)(x-8)}{8} \times \frac{-6}{6}$$

$$= x-3 + (x^2-10x+24)\frac{3}{8} + \frac{x^3-18x^2+104x-192}{8} \times -1$$

$$= \frac{1}{8}\left(8x-24+3x^2-30x+72+(-x^3+18x^2-104x+192)\right)$$

$$= \frac{1}{8}\left(-x^3+21x^2-126x+240\right)$$

$$y(5) = \frac{1}{8}\left(-(5)^3+21(5)^2-126(5)+240\right) = 1.25$$

Backward Interpolation:

Here $x_n = 10$; $y_n = 10$; $h = 2$.

$u = \frac{x-10}{2}$

$$y(x) = y_n + \frac{u}{1!}\nabla y_n + \frac{u(u+1)}{2!}\nabla^2 y_n + \frac{u(u+1)(u+2)}{3!}\nabla^3 y_n$$

$$= 10 + \left(\frac{x-10}{2}\right)(2) + \left(\frac{x-10}{2}\right)\left(\frac{x-10}{2}+1\right)\frac{(-3)}{2!} +$$

$$\left(\frac{x-10}{2}\right)\left(\frac{x-10}{2}+1\right)\left(\frac{x-10}{2}+2\right)\frac{(-6)}{3!}$$