



## DEPARTMENT OF MATHEMATICS

### UNIT-IV INTERPOLATION, NUMERICAL DIFFERENTIATION & INTEGRATION

DERIVATIVES FROM DIFFERENCE TABLES - DIVIDED DIFFERENCES AND FINITE DIFFERENCES :

$$\frac{dy}{dx} = \frac{1}{h} \left[ \Delta y_0 + \frac{2u-1}{2!} \Delta^2 y_0 + \frac{3u^2-6u+2}{3!} \Delta^3 y_0 + \frac{4u^3-18u^2+22u-6}{4!} \Delta^4 y_0 + \dots \right]$$

putting  $x = x_0$ , then  $u=0$  and above eqn. reduces to

$$\left( \frac{dy}{dx} \right)_{x=x_0} = \frac{1}{h} \left[ \Delta y_0 + \frac{1}{2} \Delta^2 y_0 + \frac{1}{3} \Delta^3 y_0 + \frac{1}{4} \Delta^4 y_0 + \frac{1}{5} \Delta^5 y_0 \right]$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2} \left[ \Delta^2 y_0 + \frac{6u-6}{3!} \Delta^3 y_0 + \frac{12u^2-36u+22}{4!} \Delta^4 y_0 + \dots \right]$$

$$\left( \frac{d^2y}{dx^2} \right)_{x=x_0} = \frac{1}{h^2} \left[ \Delta^2 y_0 + \Delta^3 y_0 + \frac{11}{12} \Delta^4 y_0 + \frac{5}{6} \Delta^5 y_0 \right]$$



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$$\frac{d^3y}{dx^3} = \frac{1}{h^3} \left[ \frac{6}{3!} \Delta^3 y_0 + \frac{24u-36}{4!} \Delta^4 y_0 + \dots \right]$$

$$\left( \frac{d^3y}{dx^3} \right)_{x=x_0} = \frac{1}{h^3} \left[ \Delta^3 y_0 - \frac{3}{2} \Delta^4 y_0 + \frac{7}{4} \Delta^5 y_2 \right]$$

NEWTON'S BACKWARD DIFFERENCE FORMULA:

$$\frac{dy}{dx} = \frac{1}{h} \left[ \nabla y_n + \frac{2u+1}{2!} \nabla^2 y_n + \frac{3u^2+6u+2}{3!} \nabla^3 y_n + \frac{4u^3+18u^2+22u+6}{4!} \nabla^4 y_n + \dots \right]$$

At  $x = x_n$ ,  $u = 0$

$$\left( \frac{dy}{dx} \right)_{x=x_n} = \frac{1}{h} \left[ \nabla y_n + \frac{1}{2} \nabla^2 y_n + \frac{1}{3} \nabla^3 y_n + \frac{1}{4} \nabla^4 y_n + \dots \right]$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2} \left[ \nabla^2 y_n + \frac{6u+6}{3!} \nabla^3 y_n + \frac{12u^2+36u+22}{4!} \nabla^4 y_n + \dots \right]$$



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$$\left(\frac{d^2y}{dx^2}\right)_{x=x_0} = \frac{1}{h^2} \left[ \nabla^2 y_n + \nabla^3 y_n + \frac{11}{12} \nabla^4 y_n + \dots \right]$$

$$\frac{d^3y}{dx^3} = \frac{1}{h^3} \left[ \frac{6}{3!} \nabla^3 y_n + \frac{24 \cdot 4 + 36}{4!} \nabla^4 y_n + \dots \right]$$

$$\left(\frac{d^3y}{dx^3}\right)_{x=x_n} = \frac{1}{h^3} \left[ \nabla^3 y_n + \frac{3}{2} \nabla^4 y_n + \frac{7}{4} \nabla^5 y_n \right]$$



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① Find  $f'(3)$  and  $f''(3)$  for the following data:

$x$	3.0	3.2	3.4	3.6	3.8	4.0
$f(x)$	-14	-10.032	-5.296	-0.256	6.672	14

$x$	$y$	$\Delta y$	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
3.0	-14	3.968				
3.2	-10.032	4.736	0.768			
3.4	-5.296	5.04	0.304	-0.464		
3.6	-0.256	6.928	1.888	1.584	2.048	
3.8	6.672	7.328	0.4	-1.488	-3.072	-5.12
4.0	14					

By Newton's forward formula

$$\left(\frac{dy}{dx}\right)_{x=x_0} = \left(\frac{dy}{du}\right)_{u=0}$$

$$= \frac{1}{h} \left[ \Delta y_0 - \frac{1}{2} \Delta^2 y_0 + \frac{1}{3} \Delta^3 y_0 - \frac{1}{4} \Delta^4 y_0 + \frac{1}{5} \Delta^5 y_0 - \dots \right]$$



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Here  $h = 0.2$

$$= \frac{1}{0.2} \left[ 3.968 - \frac{1}{2} (0.768) + \frac{1}{3} (-0.464) - \frac{1}{4} (2.048) + \frac{1}{5} (-5.12) \right]$$

$$= \frac{1}{0.2} [3.968 - 0.384 - 0.1547 - 0.512 - 1.024]$$

$$= \frac{1}{0.2} [1.8933]$$

$$= 9.4665$$

$$\left( \frac{d^2 y}{dx^2} \right)_{x=x_0} = \frac{1}{h^2} \left[ \Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12} \Delta^4 y_0 - \frac{5}{6} \Delta^5 y_0 + \dots \right]$$

$$= \frac{1}{(0.2)^2} \left[ 0.768 - (-0.464) + \frac{11}{12} (2.048) - \frac{5}{6} (-5.12) \right]$$

$$= \frac{1}{0.04} [0.768 + 0.464 + 1.8773 + 4.267]$$

$$= \frac{1}{0.04} [7.3763] = 184.4075 \approx 184.41$$