SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore–35

DEPARTMENT OF MATHEMATICS

UNIT–IV INTERPOLATION, NUMERICAL DIFFERENTIATION &

INTEGRATION

NUMERICAL INTEGRATION BY TRAPEZOIDAL

TRAPEZOIDAL RULE :

$$\int_{x_0}^{x_n} y \, dx = \frac{h}{2} \left[(y_0 + y_n) + 2(y_1 + y_2 + \cdots + y_{n-1}) \right]$$

$$= \frac{h}{2} [A + 2B]$$

where A = Sum of the first & last ordinates

B = Sum of the remaining ordinates.

(1) Using trapezoidal rule, evaluate $\int_{-1}^{1} \frac{dx}{1+x^2}$ taking 8 intervals.

Soln: Gⁿ $y(x) = \frac{1}{1+x^2}$

Here $h = \frac{b-a}{n}$ where $a = -1$, $b = 1$, and $n = 8$

$\Rightarrow h = \frac{2}{8} = 0.25$

x : -1 -0.75 -0.5 -0.25 0 0.25 0.5 0.75 1

y : 0.5 0.64 0.8 0.9412 1 0.9412 0.8 0.64 0.5

Trapezoidal rule,

$$\int_{-1}^{1} \frac{1}{1+x^2}\, dx = \frac{h}{2}\left[(y_0 + y_n) + 2(y_1 + y_2 + \cdots + y_{n-1}) \right]$$

$$= \frac{h}{2}\left[\text{sum of the first and last ordinates} + 2 \times \text{sum of the remaining ordinates} \right]$$

$$= \frac{0.25}{2}\left[(0.5 + 0.5) + 2(0.64 + 0.8 + 0.9412 + 0.9412 + 0.8 + 0.64) \right]$$

$$= \frac{0.25}{2} \times 12.5248$$

$$= 1.5656$$

② Dividing the range into 10 equal parts, find the value

of $\displaystyle\int_{0}^{\pi/2} \sin x \, dx$ by (i) Trapezoidal rule

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Soln:

x :	0	$\pi/20$	$2\pi/20$	$3\pi/20$	$4\pi/20$	$5\pi/20$
$y = \sin x$:	0	0.1564	0.3090	0.4540	0.5878	0.7071

x :	$6\pi/20$	$7\pi/20$	$8\pi/20$	$9\pi/20$	$10\pi/20$
$y : \sin x$:	0.8090	0.8910	0.9511	0.9877	1

By Trapezoidal rule;

$$\int_0^{\pi/2} \sin x \, dx = \frac{h}{2}\left[(y_0 + y_{11}) + 2(y_1 + y_2 + \cdots + y_{10})\right]$$

$$\boxed{h = \frac{\frac{\pi}{2} - 0}{10} = \frac{\pi}{20}}$$

$$= \frac{h}{2}\left[(0+1) + 2(0.1564 + 0.3090 + 0.4540 \right.$$
$$+ 0.5878 + 0.7071 + 0.8090 +$$
$$\left. 0.8910 + 0.9511 + 0.9877)\right]$$

$$= \frac{\pi}{20} \cdot \frac{1}{2}\left[12.7062\right]$$

$$= 0.9980$$