



DEPARTMENT OF MATHEMATICS

UNIT-V NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONS

MILNE'S PREDICTOR AND CORRECTOR METHODS

FORMULAS:

$$y_{n+1,P} = y_{n-3} + \frac{4h}{3} [2y'_{n-2} - y'_{n-1} + 2y'_n] \rightarrow \text{Milne's predictor formula}$$

$$y_{n+1,C} = y_{n-1} + \frac{h}{3} [y'_{n-1} + 4y'_n + y'_{n+1}] \rightarrow \text{Milne's corrector formula}$$

Solve $y' = x - y^2$, $0 \leq x \leq 1$, $y(0) = 0$, $y(0.2) = 0.02$, $y(0.4) = 0.0795$, $y(0.6) = 0.1762$ by Milne's method to find $y(0.8)$ and $y(1)$.

Soln:

Givn:

$$x_0 = 0 \rightarrow y_0 = 0$$

$$x_1 = 0.2 \rightarrow y_1 = 0.02$$

$$x_2 = 0.4 \rightarrow y_2 = 0.0795$$

$$x_3 = 0.6 \rightarrow y_3 = 0.1762$$

$$x_4 = 0.8 \rightarrow y_4 = ?$$

$$x_5 = 1 \rightarrow y_5 = ?$$

Here $h = 0.2$.



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Wkt Milne's predictor formula is

$$y_{n+1, p} = y_{n-3} + \frac{4h}{3} [2y'_{n-2} - y'_{n-1} + 2y'_n]$$

$$y_{4, p} = y_0 + \frac{4h}{3} [2y'_1 - y'_2 + 2y'_3]$$

Gn. $y' = x - y^2$

$$y'_1 = x_1 - y_1^2 = 0.2 - (0.02)^2 = 0.1996$$

$$y'_2 = x_2 - y_2^2 = 0.4 - (0.0795)^2 = 0.3937$$

$$y'_3 = x_3 - y_3^2 = 0.6 - (0.1762)^2 = 0.5690$$

$$\begin{aligned} y_{4, p} &= 0 + \frac{4(0.2)}{3} [2 \times 0.1996 - 0.3937 + 2 \times 0.5690] \\ &= 0.3049 \end{aligned}$$

$$y'_4 = x_4 - y_4^2 = 0.8 - (0.3049)^2 = 0.707$$



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$$\begin{aligned}y_{4,c} &= y_2 + \frac{h}{3} [y_2' + 4y_3' + y_4'] \\&= 0.0795 + \frac{0.2}{3} [0.3937 + 4 \times 0.5690 + 0.707] \\&= 0.3046\end{aligned}$$

∴ Corrected value of y at $x = 0.8$ is 0.3046 .

To find $y(1)$:

$$\begin{aligned}y_{5,p} &= y_1 + \frac{4h}{3} [2y_2' - y_3' + 2y_4'] \\&= 0.02 + 4 \times \frac{0.2}{3} [2 \times 0.3937 - 0.5690 + 2 \times 0.707] \\&= 0.4553\end{aligned}$$

$$y_5' = x_5 - y_5^2 = 1 - (0.4553)^2 = 0.7327$$

$$\begin{aligned}y_{5,c} &= y_3 + \frac{h}{3} [y_3' + 4y_4' + y_5'] \\&= 0.1762 + \frac{0.2}{3} [0.569 + 4 \times 0.707 + 0.7327] \\&= 0.4515\end{aligned}$$

∴ Corrected value of y at $x = 1$ is 0.4515 .



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① using Milne's method find $y(4.4)$ for $5xy' + y^2 - 2 = 0$
given $y(4) = 1$, $y(4.1) = 1.0049$, $y(4.2) = 1.0097$ and
 $y(4.3) = 1.0143$.

Soln: $y_{4,p} = 1.01897$; $y_{4,c} = 1.01874$

② using Runge-Kutta method calculate $y(0.1)$, $y(0.2)$ and
 $y(0.3)$ for $\frac{dy}{dx} - \frac{2xy}{1+x^2} = 1$, $y(0) = 0$. Taking these
values as starting values find $y(0.4)$ by Milne's method

Soln: $y(0.1) = 0.1006$
 $y(0.2) = 0.2052$
 $y(0.3) = 0.3176$
 $y(0.4) = 0.4413$

} By Rk method.