SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)
Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai
Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &
Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

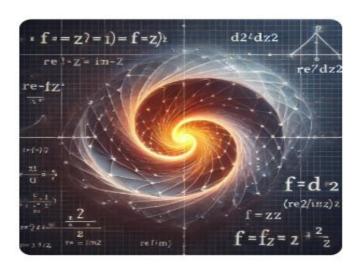## Puzzle: The Mysterious Holomorphic Function

A mysterious function $f(z)$ is holomorphic in the entire complex plane and satisfies the condition:

$$\left(\frac{d}{dz}\right)^2 f(z) = f(z)$$

Additionally, it is given that:

$$f(0) = 2, \quad f'(0) = 3.$$

◆ Question: Can you determine the explicit form of $f(z)$?

Given Differential Equation:

$$\frac{d^2}{dz^2} f(z) = f(z)$$

with initial conditions:

$$f(0) = 2, \quad f'(0) = 3.$$

Step 1: Solve the Differential Equation

This is a **linear homogeneous differential equation** with constant coefficients. The characteristic equation is:

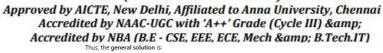$$r^2 - 1 = 0$$

Solving for r:

$$r = \pm 1.$$

Thus, the **general solution** is:

Ask anything

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Thus, the **general solution** is:

$$f(z) = Ae^z + Be^{-z}.$$

Step 2: Apply Initial Conditions

Condition 1: $f(0) = 2$

Substituting $z = 0$ into the general solution:

$$Ae^0 + Be^0 = A + B = 2.$$

Condition 2: $f'(0) = 3$

First, differentiate $f(z)$:

$$f'(z) = Ae^z - Be^{-z}.$$

Substituting $z = 0$:

$$Ae^0 - Be^0 = A - B = 3.$$

Step 3: Solve for A and B

We have the system of equations:

1. $A + B = 2$

2. $A - B = 3$

Adding both equations:

$$2A = 5 \quad \Rightarrow \quad A = \frac{5}{2}.$$

Subtracting the second equation from the first:

$$2B = -1 \quad \Rightarrow \quad B = \frac{1}{2}.$$

Final Answer:

$$f(z) = \frac{5}{2}e^z + \frac{1}{2}e^{-z}.$$

Alternatively, using hyperbolic functions:

$$f(z) = 3\cosh z + \sinh z.$$

Thus, the explicit function satisfying the given ODE and initial conditions is:

$$f(z) = 3\cosh z + \sinh z.$$