Transformations in OpenGL

- Translate
- Rotate
- Scale

- Push Matrix
- Pop Matrix

OpenGL Functions

- Transformations in OpenGL are not drawing commands. They are retained as part of the graphics state.
- When drawing commands are issued, the current transformation is applied to the points drawn.
- Transformations are cumulative.

Translation

Offset (tx, ty, tz) is applied to all subsequent coordinates. Effectively moves the origin of coordinate system.

- $x' = x + \mathrm{tx}\ , \quad y' = y + \mathrm{ty}, \quad z' = z + \mathrm{tz}$
- OpenGL function is `glTranslate`
- `glTranslatef`(tx, ty, tz);

Rotation

Expressed as rotation through angle θ about an axis direction (x,y,z) .

- OpenGL function – `glRotatef` (θ, x,y,z). So
  `glRotatef`(30.0, 0.0, 1.0, 0.0)
  rotates by 30° about y-axis.
- Note carefully:
  - `glRotate` wants angles in degrees.
  - C math library (sin, cos etc.) wants angles in radians.
  - $degs = rads * 180/\pi$; $rads = degs * \pi / 180$
- Positive angle? Right hand rule: if the thumb points along the vector of rotation, a positive angle has the fingers curling towards the palm.

Rotation (cont.)

- Frequently the axis is one of the coordinate axes. Common terms:
  - rotation about y-axis is heading/yaw
  - rotation about x-axis is pitch/elevation
  - rotation about z-axis is roll/bank
- 3-d rotation is an extremely difficult topic! There are several different mathematical formulations. Rotations do not commute – the order that transformations are done matters.

Scaling

- Multiply subsequent coordinates by scale factors sx, sy, sz. (Note: these are not a point, not a vector, just 3 numbers)

  $x' = \mathrm{sx} * x\ ,\quad y' = \mathrm{sy} * y,\quad z' = \mathrm{sz} * z$
- Often sx = sy = sz for a *uniform* scaling effect. If the factors are different, the scaling is called *anamorphic*.
- OpenGL function – `glScale` For example,

  ```
  glScalef(0.5,0.5,0.5);
  ```

  would cause all objects drawn subsequently to be half as big.

Order of transformations

- Transformations are cumulative and the order matters:
  - The sequence
    1. Scale 2, 2, 2
    2. Translate by (10, 0, 0)

    will scale subsequent objects by factor of 2 about an origin that is 20 along the *x*-axis
  - The sequence
    1. Rotate 90.0 deg about (0, 1, 0)
    2. Translate by (10, 0, 0)

    will set an origin 10 along the –ve *z*-axis
- For each object, the usual sequence is:
  1. Translate (move the origin to the right location)
  2. Rotate (orient the coordinate axes right)
  3. Scale (get the object to the right size)

Matrix representation

- Every 3-d point can be written as a 4-element vector and every 3-d transformation as a 4x4 matrix. (Yes, FOUR)
- For a point P (x,y,z), a fourth 'dummy' coordinate is appended. Internally the graphics card will treat the point as having 4 elements ($x,y,z,1$). These are called the *homogeneous coordinates* of P. (More on this later.)
- The identity matrix leaves any original vector unchanged:

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Matrix representation

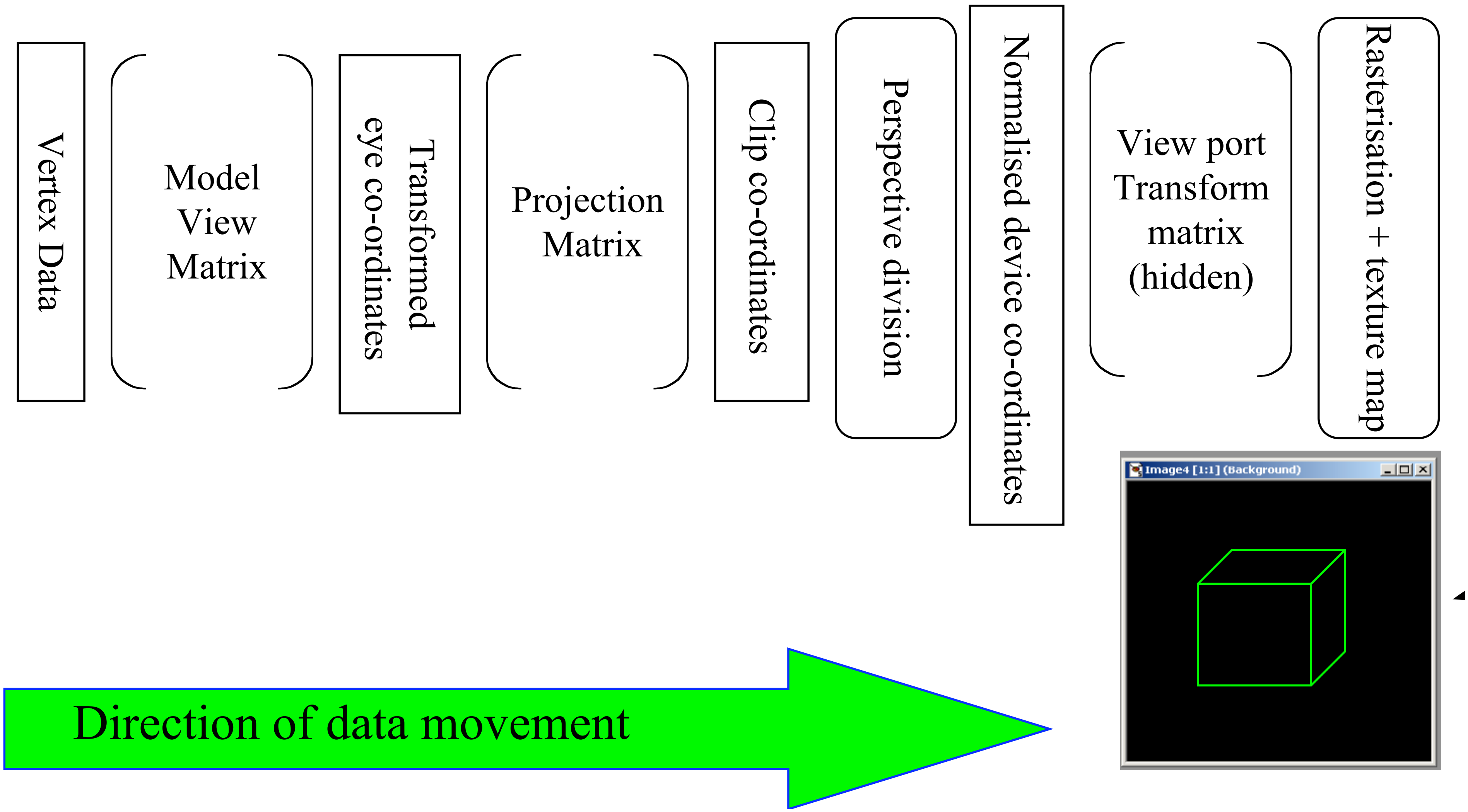
- If points are represented by column vectors, a translation is represented by a matrix with the offset values in the 4th column:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & tx \\ 0 & 1 & 0 & ty \\ 0 & 0 & 1 & tz \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

•A rotation matrix uses the top left 3x3 area. A scaling matrix puts the scale factors on the diagonal.

•A matrix can represent *any* 3-d transformation, including some we haven't covered such as shearing and perspective projection.

The OpenGL pipeline



Matrices and Graphics State

- Each of the transformations above (Model View Matrix, Projection Matrix etc.) is maintained by OpenGL as part of the graphics state. (Current Transformation Matrix CTM)
- `glLoadIdentity` sets the CTM to the identity matrix, for a "fresh start".
- When `glRotate` or similar command is issued, the appropriate transformation matrix is updated.
- Note carefully that the rotation matrix doesn't overwrite the old CTM. It updates CTM by matrix multiplication.
- In fact the CTM is so important that OpenGL can keep several of them in a stack. By popping the stack, you can recover an old and possibly still-useful CTM.

Nested Transformations

- The sequence
  translate 1.5 0 0
  cube
  translate 8.5 0 0
  cube
  will draw two cubes with x centres 1.5 and 10.0 respectively.
- We could create the same image with the sequence
  save state
  translate 1.5 0 0
  cube
  restore state
  save state
  translate 10.0 0 0
  cube
  restore state

- Here both cubes have an absolute translation and the order in which the two cubes are drawn does not matter.

Push and Pop

- `glMatrixMode(GL_MODELVIEW)`
- `glMatrixMode(GL_PROJECTION)`
- `glMatrixMode(GL_TEXTURE)`

- **`glPushMatrix`**`();`
  - Save the state.
  - Push a copy of the CTM onto the stack.
  - The CTM itself is unchanged.
- **`glPopMatrix`**`();`
  - Restore the state, as it was at the last Push.
  - Overwrite the CTM with the matrix at the top of the stack.
- **`glLoadIdentity`**();
  - Overwrite the CTM with the identity matrix.

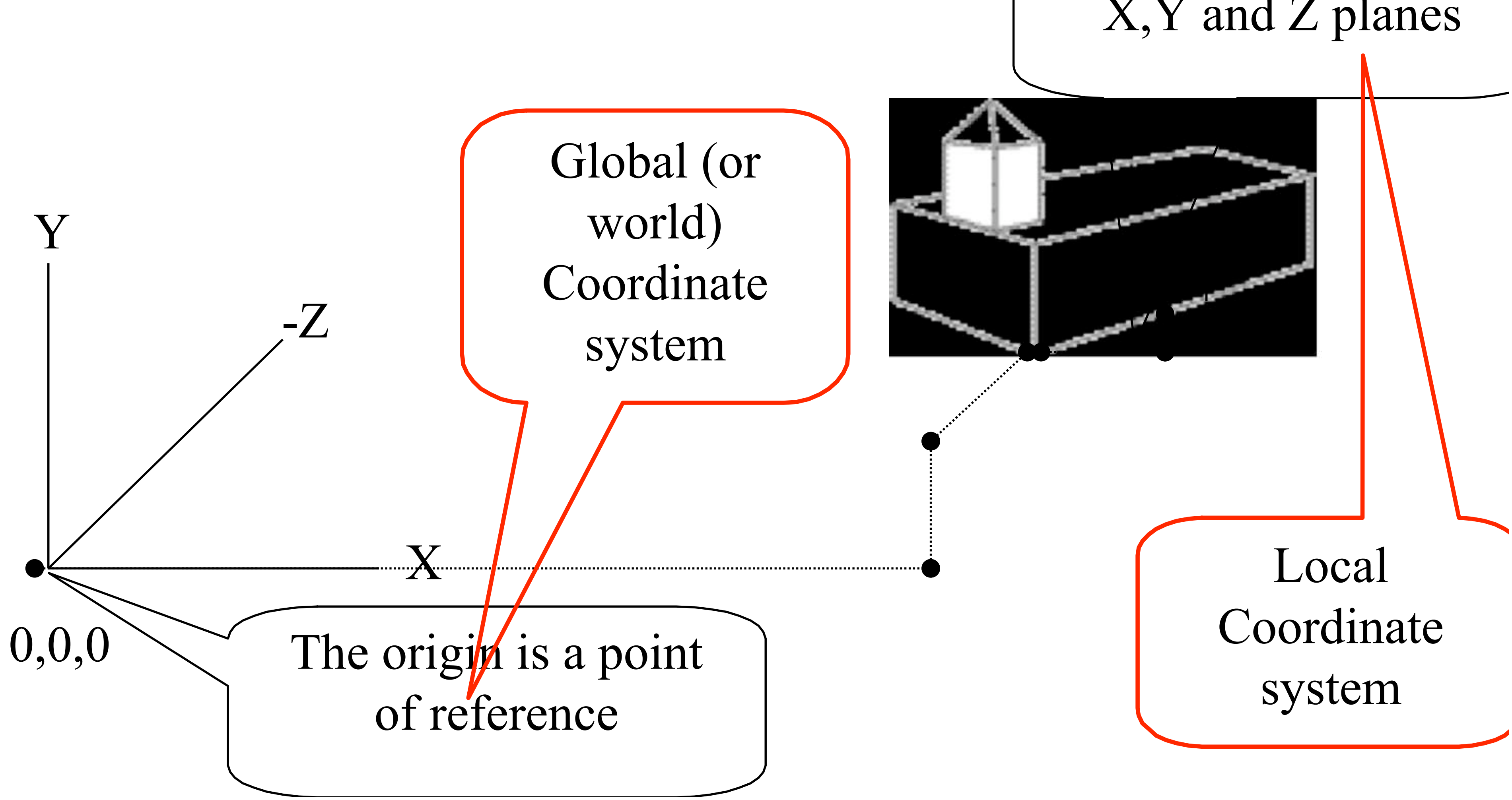
Local coordinate system

- The standard way to construct a complex 3D model is to define each individual part in a local coordinate system. This has whatever origin, and whatever unit, is most convenient.
- Typically we will draw the part centred at the origin, and aligned with the coordinate axes.
- Each part is then transformed relative to some parent before being rendered. Groups of parts may themselves have a parent, and so on up to the final world coordinate system.
- There are many changes of CTM, so many that your head will spin for a couple of weeks. Everyone has to get skilled at this – it's the source of the power and flexibility and (believe it or not) ease of use of a graphics system.

Coordinate systems

So the trick to understanding world co ordinates is to know what the reference point is (and it relation to the origin)

The <u>world</u> coordinate system is the one in which the entire 3D model is defined.



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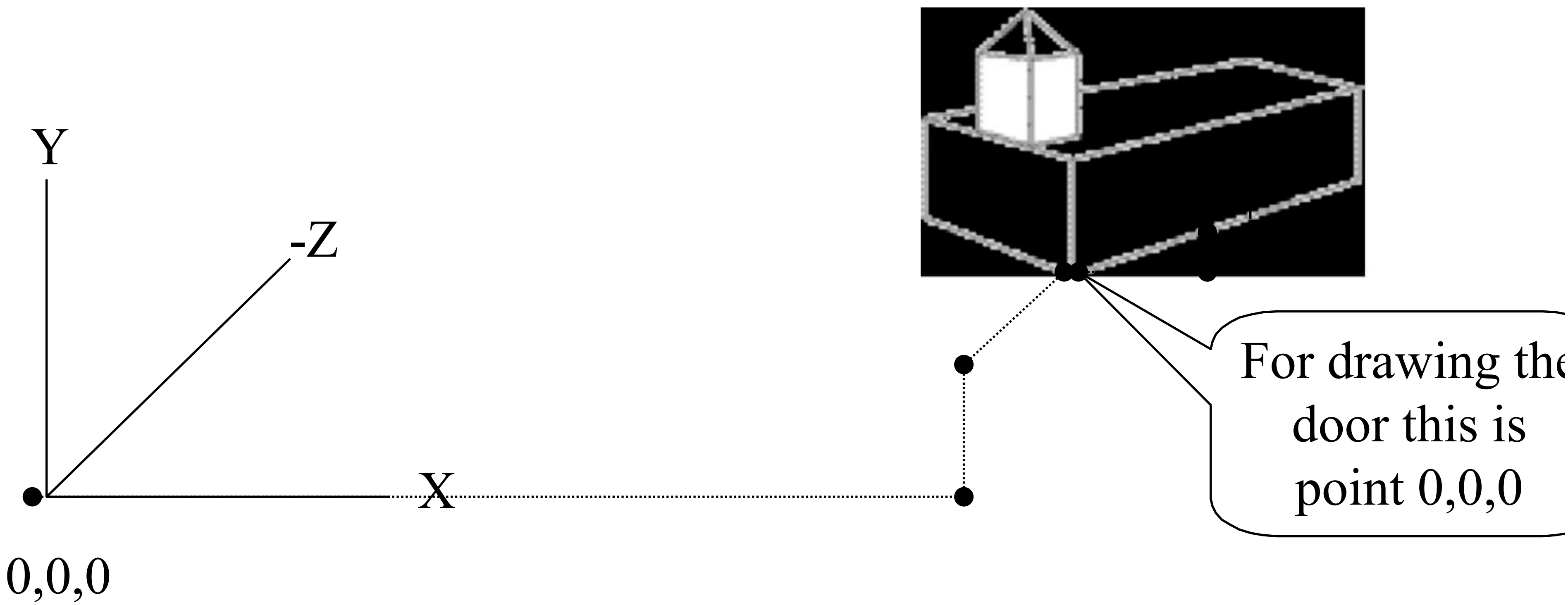
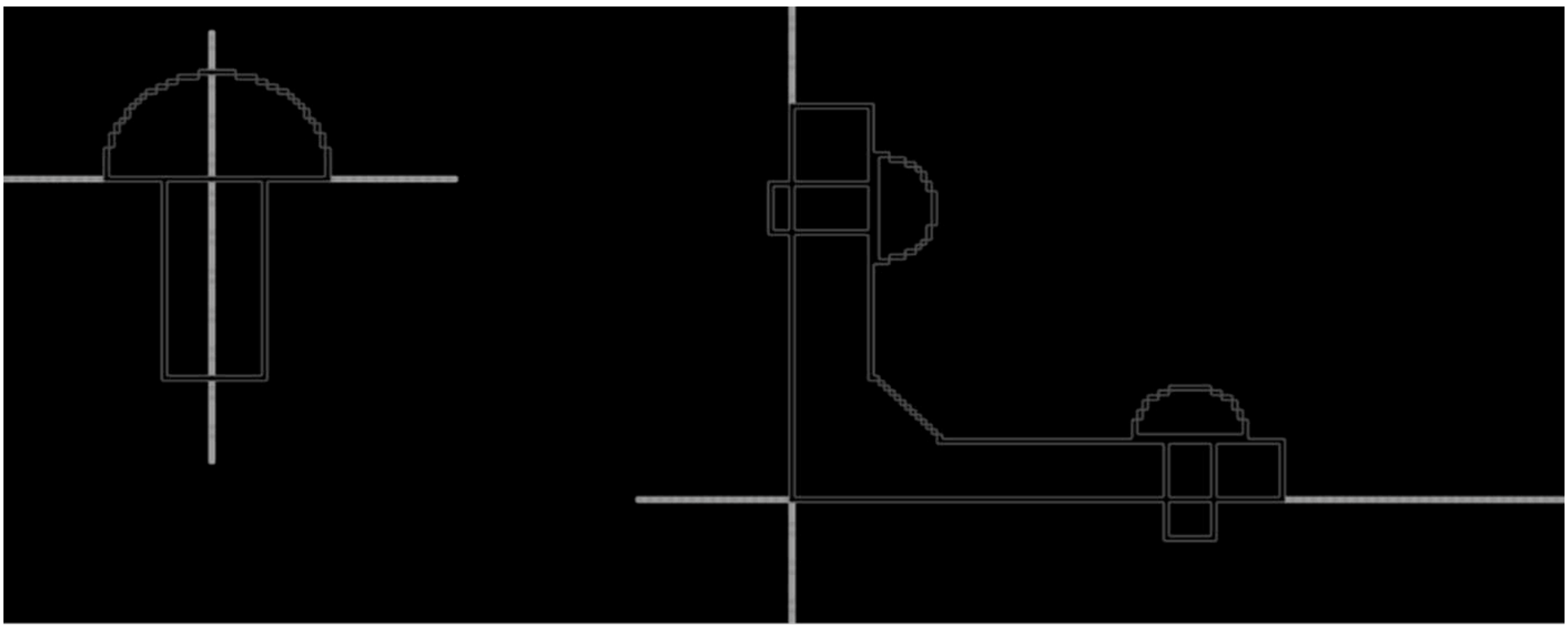


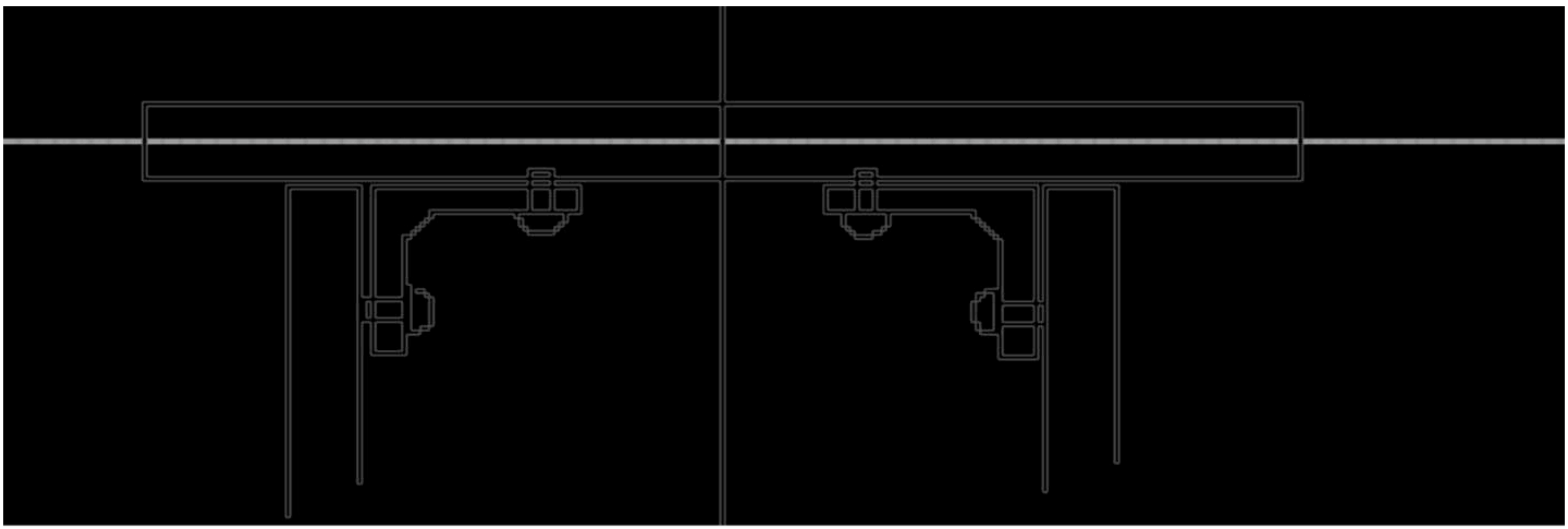
Scene Graphs

- We define a rivet in a local coordinate system, and then translate and rotate each rivet within the coordinate system of a bracket:



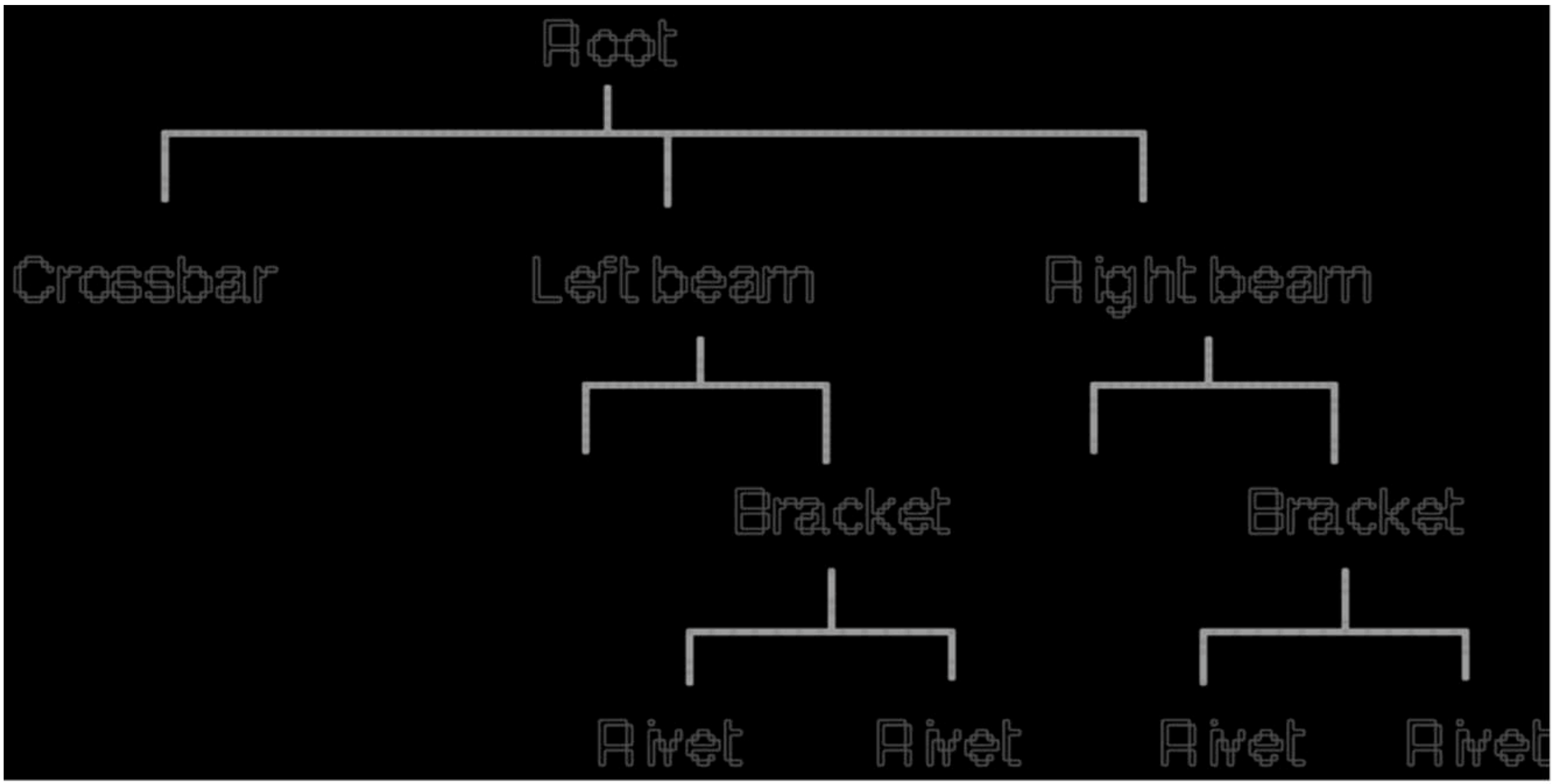
Scene Graphs (cont.)

- Each bracket is in turn translated and rotated within the coordinate system of a subassembly:



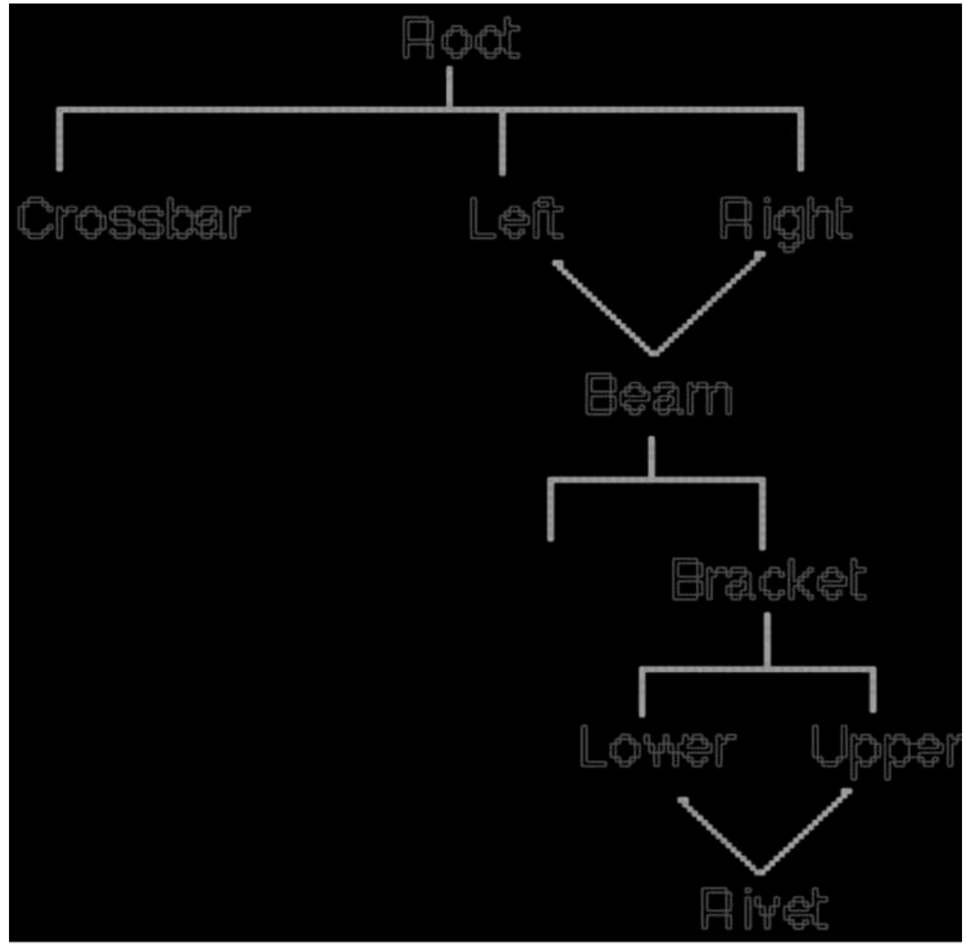
Representation as Graph

- This hierarchical 3D model can be represented as a scene graph:



DAG, Directed Acyclic Graph

- Each node in the scene graph inherits the CTM from its parent. Descending the graph pushes the matrix stack, ascending pops.
- Above, the scene graph was shown as a rooted tree. But a scene graph is better represented as a DAG, directed acyclic graph, in which each node can have more than one predecessor. This picture more accurately reflects the structure of our function calls.



DAG, Directed Acyclic Graph

- May be implicit in Code
  - In the structure of your calls to drawing functions e.g. `drawCar`(), `drawAxle`(), `drawWheel`(), `drawCylinder`()
- Can be in data
  - Such as a mesh (eg .3DS format)
- Can be in a human readable language
  - VRML

VRML
A Scene Graph Language

- Virtual Reality Modelling Language
  - I think Virtual Reality Mark up Language which is sometimes seen is wrong
- It is a scene graph language
- Designed by SGI and is closely related to OpenGL (many features in common)
- Despite early successes VRML has not really caught on
- Some companies are trying to future proof their character meshes by storing them in VRML
- Can be useful to us because it is human readable an relatively easy to understand