Smoothing frequency domain filters

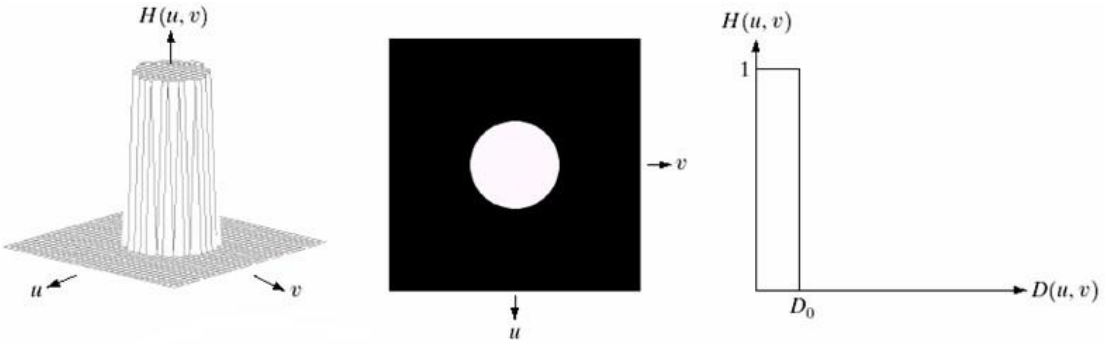
Ideal Lowpass Filter (ILPF)

ILPF is the simplest lowpass filter that "cuts off" all high frequency components of the DFT that are at a distance greater than a specified distance D_0 from the origin of the (centered) transform.

The transfer function of this filter is:

$$H(u,v) = \begin{cases} 1 & if \;\; D(u,v) \le D_0 \\ 0 & if \;\; D(u,v) > D_0 \end{cases}$$

where D_0 is the cutoff frequency, and $D(u,v) = \sqrt{(u - M/2)^2 + (v - N/2)^2}$

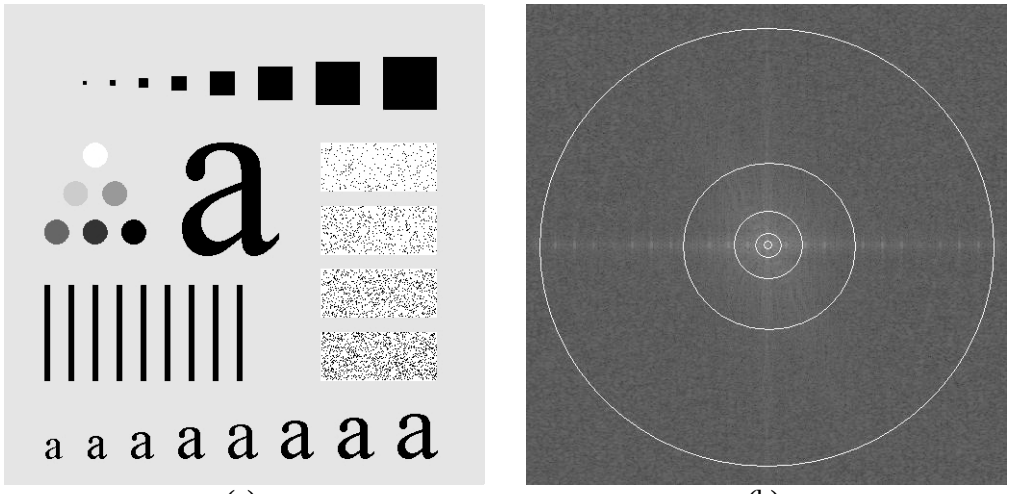


(a) (b) (c)

Figure 7.1 (a) Ideal lowpass filter. (b) ILPF as an image. (c) ILPF radial cross section

The ILPF indicates that all frequencies inside a circle of radius D_0 are passed with no attenuation, whereas all frequencies outside this circle are completely attenuated.

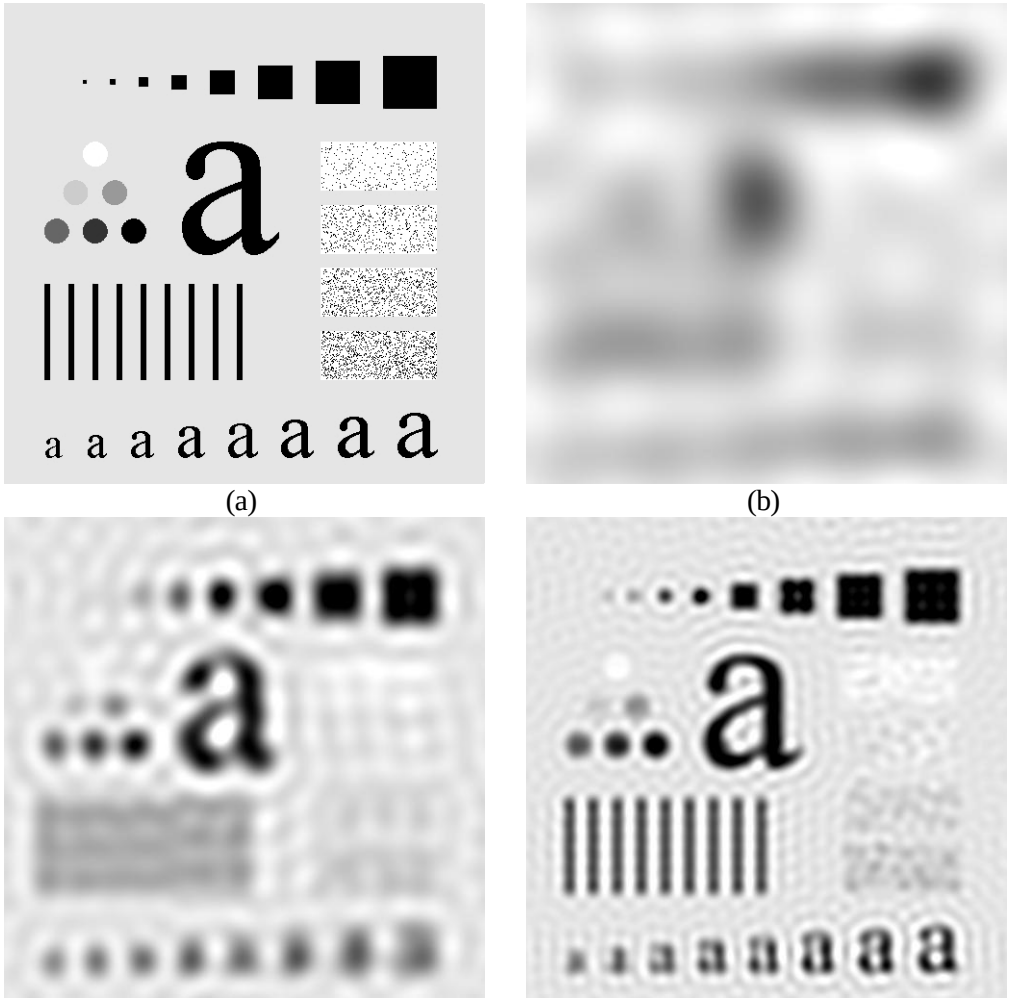
The next figure shows a gray image with its Fourier spectrum. The circles superimposed on the spectrum represent cutoff frequencies 5, 15, 30, 80 and 230.



(a) (b)

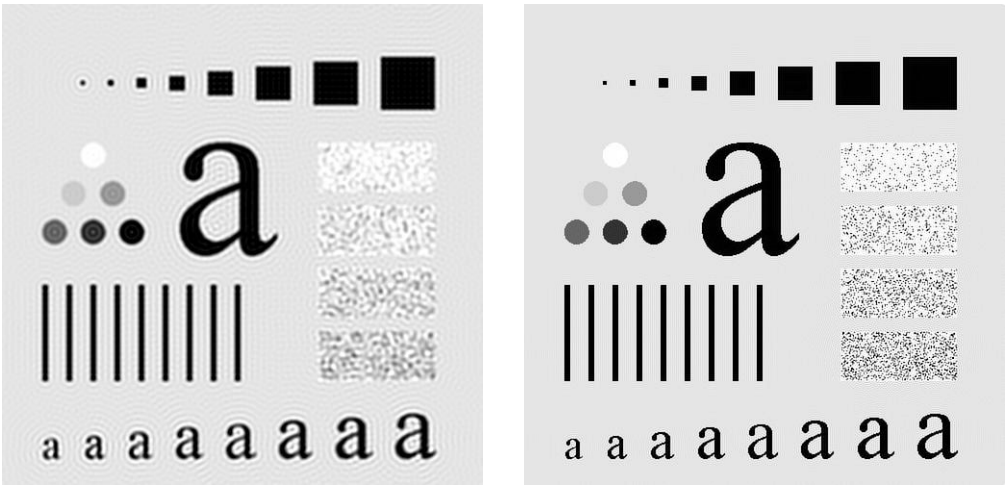
Figure 7.2 (a) Original image. (b) its Fourier spectrum

The figure below shows the results of applying ILPF with the previous cutoff frequencies.



(a) (b)

(c) (d)



(e) (f)

Figure 7.3 (a) Original image. (b) - (f) Results of ILPF with cutoff frequencies 5, 15, 30, 80, and 230 respectively.

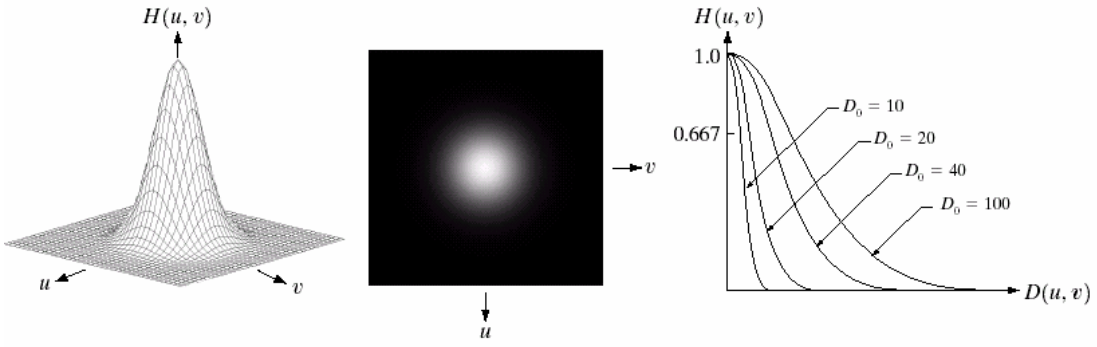
We can see the following effects of ILPF:

1. Blurring effect which decreases as the cutoff frequency increases.
2. Ringing effect which becomes finer (i.e. decreases) as the cutoff frequency increases.

Gaussian Lowpass Filter (GLPF)

The GLPF with cutoff frequency D_0 is defined as:

$$H(u,v) = e^{-D^2(u,v)/2D_0{}^2}$$

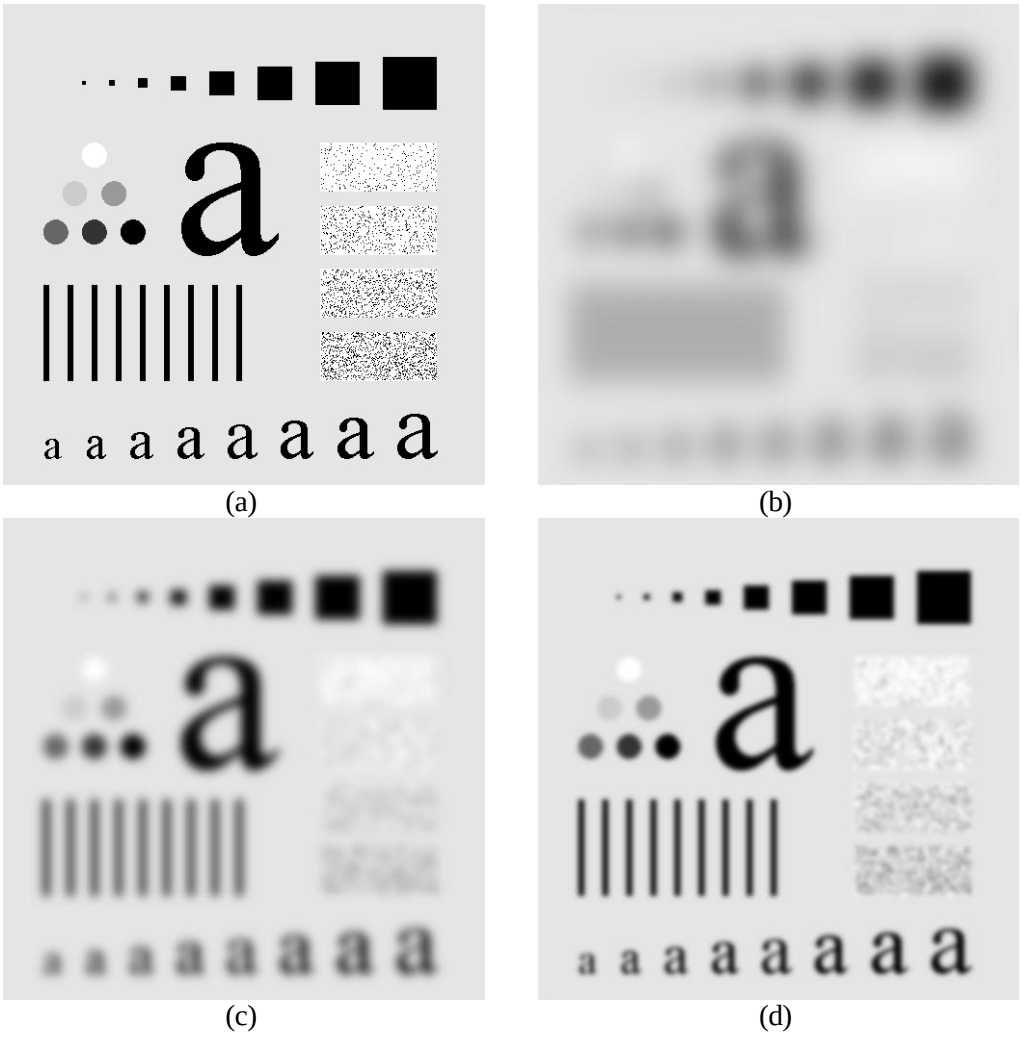


(a) (b) (c)

Figure 7.4 (a) Gaussian lowpass filter. (b) GLPF as an image. (c) GLPF radial cross section

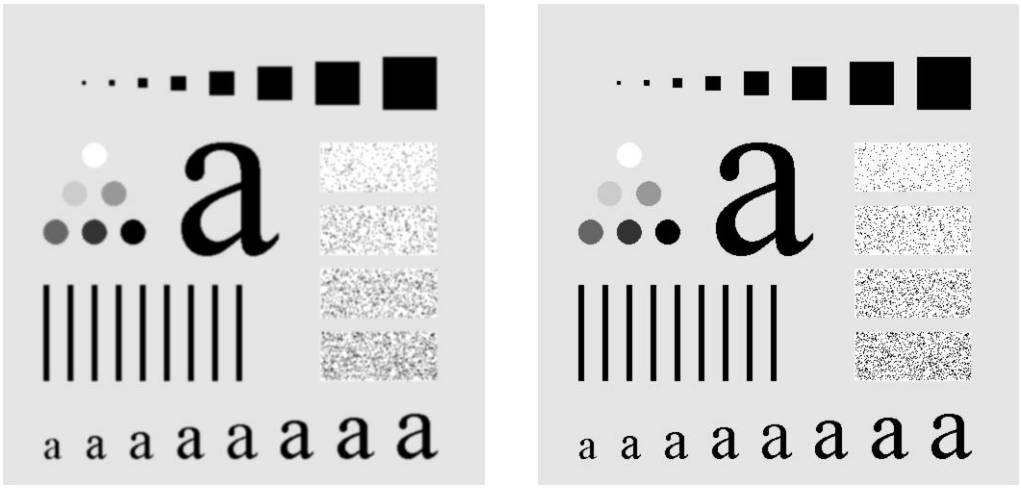
Unlike ILPF, the GLPF transfer function does not have a sharp transition that establishes a clear cutoff between passed and filtered frequencies. Instead, GLPF has a smooth transition between low and high frequencies.

The figure below shows the results of applying GLPF on the image in Figure 7.2(a) with the same previous cutoff frequencies.



(a) (b)

(c) (d)



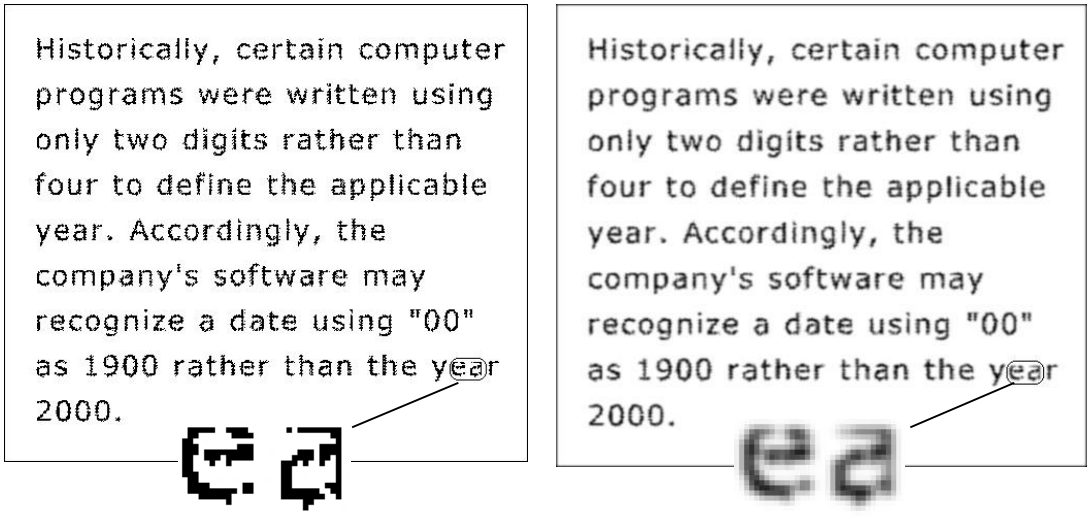
(e) (f)

Figure 7.5 (a) Original image. (b) - (f) Results of GLPF with cutoff frequencies 5, 15, 30, 80, and 230 respectively.

We can see the following effects of GLPF compared to ILPF:

1. Smooth transition in blurring as a function of increasing cutoff frequency.
2. No ringing effect.

Smoothing (lowpass) filtering is useful in many applications. For example, GLPF can be used to bridge small gaps in broken characters by blurring it as shown below. This is useful for character recognition.



(a) (b)

Figure 7.6 (a) Text of poor resolution. (b) Result of applying GLPF with cutoff=80 on (a)

GLPF can also be used for cosmetic processing prior to printing and publishing as shown below.

(a) (b)

Figure 7.7 (a) Original image. (b) Result of filtering with GLPF with cutoff=80

Sharpening frequency domain filters

Edges and sudden changes in gray levels are associated with high frequencies. Thus to enhance and sharpen significant details we need to use highpass filters in the frequency domain

For any lowpass filter there is a highpass filter:

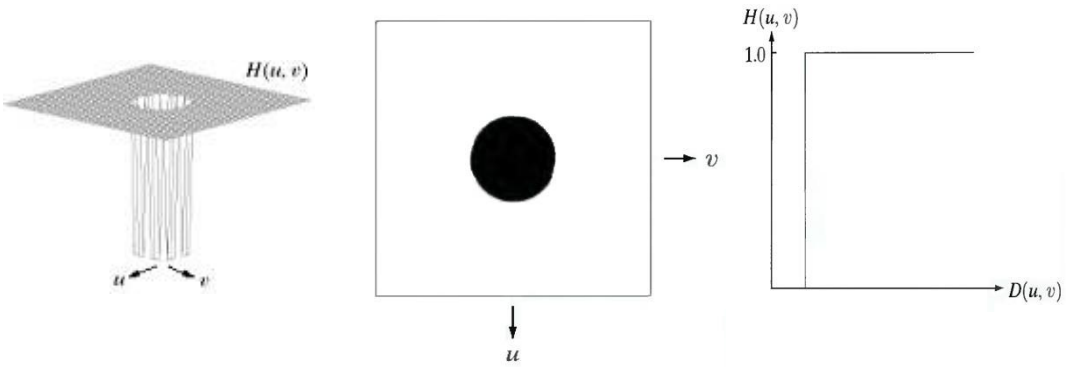
$$H_{hp}(u,v) = 1 - H_{lp}(u,v)$$

Ideal Highpass Filter (IHPF)

The IHPF cuts off all low frequencies of the DFT but maintain the high ones that are within a certain distance from the center of the DFT.

$$H(u,v) = \begin{cases} 1 & if \;\; D(u,v) > D_0 \\ 0 & if \;\; D(u,v) \leq D_0 \end{cases}$$

where D_0 is the cutoff frequency, and $D(u,v) = \sqrt{(u - M/2)^2 + (v - N/2)^2}$

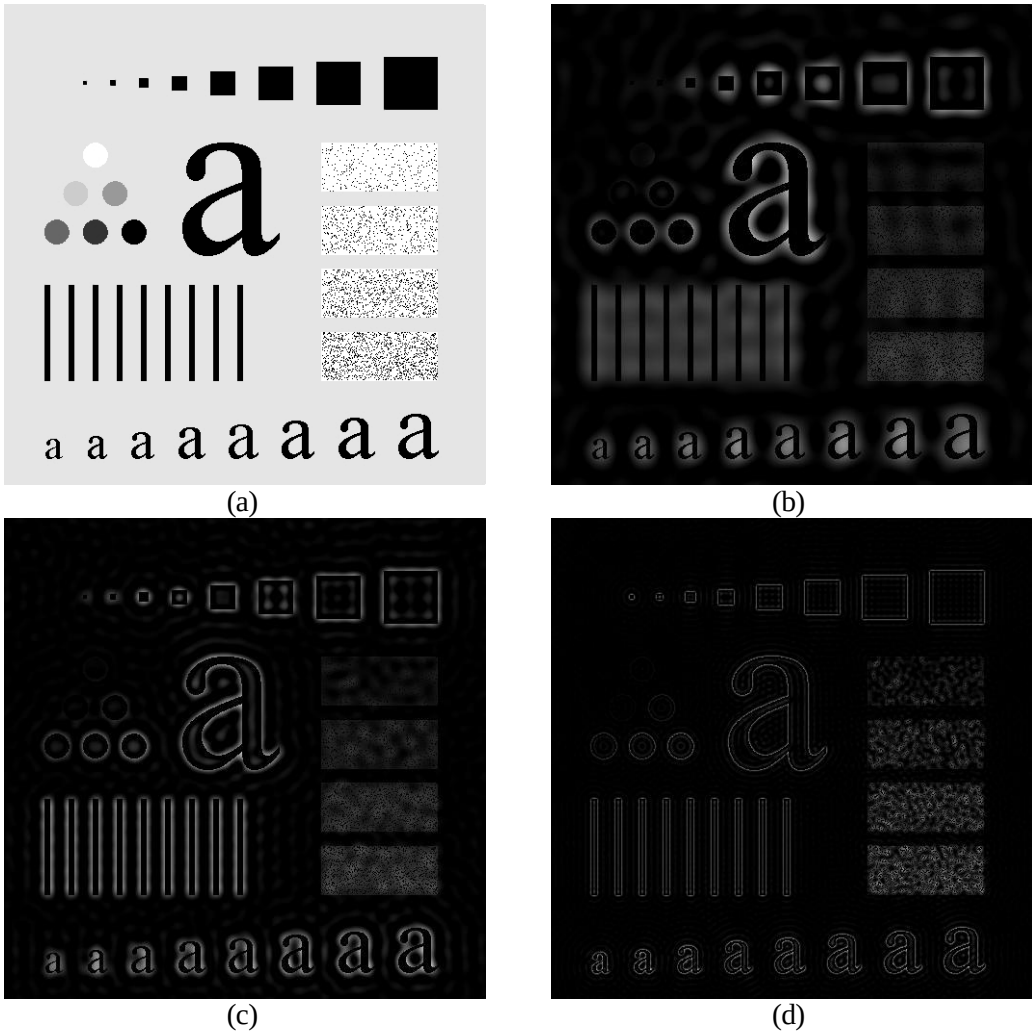


(a) (b) (c)

Figure 7.8 (a) Ideal highpass filter. (b) IHPF as an image. (c) IHPF radial cross section

The IHPF sets to zero all frequencies inside a circle of radius D_0 while passing, without attenuation, all frequencies outside the circle.

The figure below shows the results of applying IHPF with cutoff frequencies 15, 30, and 80.



(a) (b) (c) (d)

Figure 7.9 (a) Original image. (b) - (d) Results of IHPF with cutoff frequencies 15, 30, and 80 respectively.

We can see the following effects of IHPF:

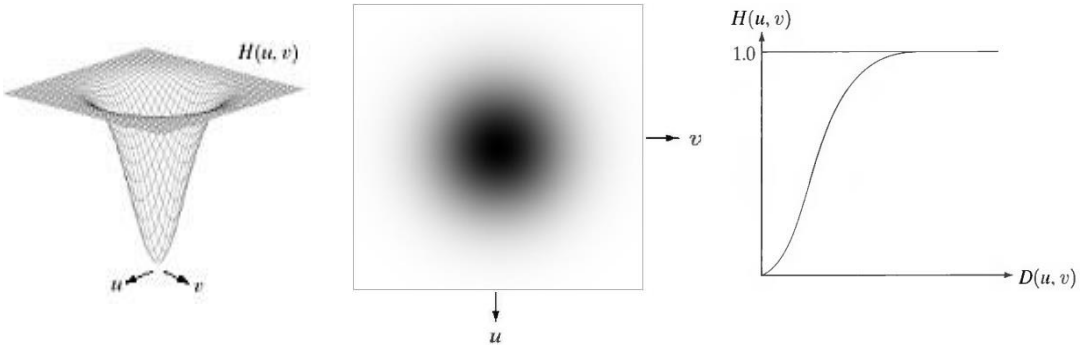
1. Ringing effect.
2. Edge distortion (i.e. distorted, thickened object boundaries).

Both effects are decreased as the cutoff frequency increases.

Gaussian Highpass Filter (GHPF)

The Gaussian Highpass Filter (GHPF) with cutoff frequency at distance D_0 is defined as:

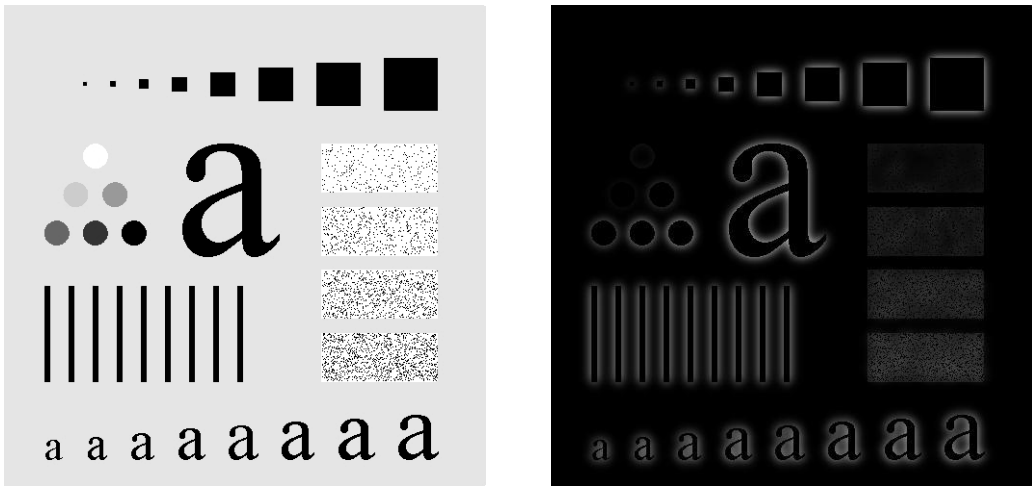
$$H(u,v) = 1 - e^{-D^2(u,v)/2D_0{}^2}$$



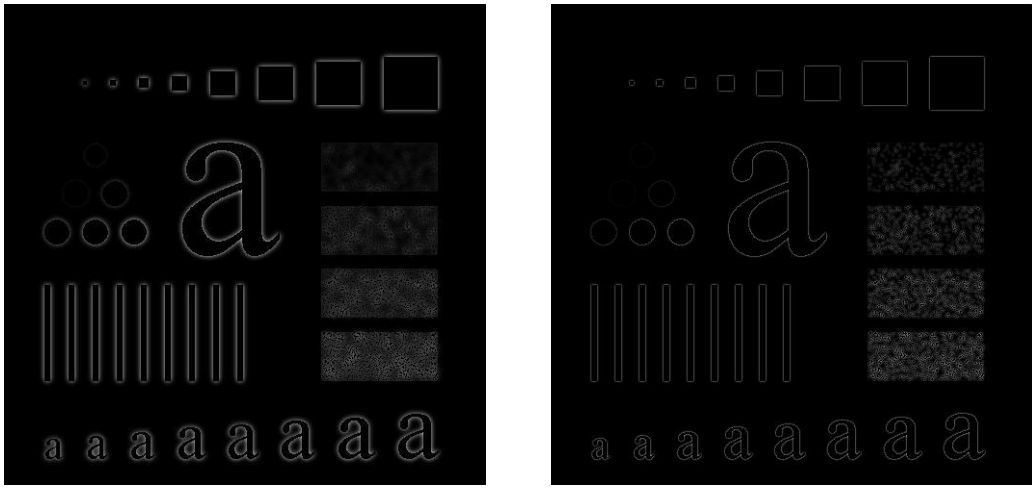
(a) (b) (c)

Figure 7.10 (a) Gaussian highpass filter. (b) GHPF as an image. (c) GHPF radial cross section

The figure below shows the results of applying GHPF with cutoff frequencies 15, 30 and 80.



(a) (b)



(c) (d)

Figure 7.11 (a) Original image. (b) - (d) Results of GHPF with cutoff frequencies 15, 30, and 80 respectively.

The effects of GHPF in comparison with IHPF are:

1. No ringing effect.
2. Less edge distortion.
3. The results are smoother than those obtained by IHPF.