DEPARTMENT OF MATHEMATICS

UNIT–IV INTERPOLATION, NUMERICAL DIFFERENTIATION AND NUMERICAL INTEGRATION

PART A

1.State Lagrange's interpolation formula.

Solution:

Let y=f(x) be a function which takes the values $y_0, y_1, y_2, \ldots, y_n$ corresponding to x= $x_0, x_1, x_2, \ldots, x_n$.
Then,Lagrange's interpolation formula is

$$y = f(x) = \frac{(x-x_1)(x-x_2)\ldots\ldots(x-x_n)}{(x_0-x_1)(x_0-x_2)\ldots\ldots(x_0-x_n)} y_0 + \frac{(x-x_0)(x-x_2)\ldots\ldots(x-x_n)}{(x_1-x_0)(x_1-x_2)\ldots\ldots(x_1-x_n)} y_1 + \ldots\ldots$$
$$+ \frac{(x-x_0)(x-x_1)\ldots\ldots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\ldots\ldots(x_n-x_{n-1})} y_n$$

2.What is Inverse interpolation?

Solution:

Inverse interpolation is the process of finding the value of x corresponding to a value of y,not present in the table.

3.Give the inverse of lagrange's interpolation formula.

Solution:

$$x = \frac{(y-y_1)(y-y_2)\ldots\ldots(y-y_n)}{(y_0-y_1)(y_0-y_2)\ldots\ldots(y_0-y_n)} x_0 + \frac{(y-y_0)(y-y_2)\ldots\ldots(y-y_n)}{(y_1-y_0)(y_1-y_2)\ldots\ldots(y_1-y_n)} x_1 + \ldots\ldots$$
$$+ \frac{(y-y_0)(y-y_1)\ldots\ldots(y-y_{n-1})}{(y_n-y_1)(y_n-y_1)\ldots\ldots(y_n-y_{n-1})} x_n$$

4.What is the advantage of Lagrange's formula?

Solution:Lagrange's interpolation formula can be used whether the values of x,the independent variable are equally spaced or not whether the difference of y become smaller or not.

5.Define Divied difference.

Solution:

The first divided difference of f(x) for the arguments x_0, x_1 is

$$\underset{(\Delta)}{f(x_0, x_1)} = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

similarly, $\underset{\Delta}{f(x_1, x_2)} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$ and so on.

The second divided difference of f(x) for three arguments x_0, x_1, x_2 is

$$\underset{(\Delta^2)}{f(x_0, x_1, x_2)} = \frac{f(x_1, x_2) - f(x_0, x_1)}{x_2 - x_0}$$

$$\underset{(\Delta^2)}{f(x_1, x_2, x_3)} = \frac{f(x_2, x_3) - f(x_1, x_2)}{x_3 - x_1}$$ and so on.

6. Show that $\underset{bcd}{\Delta^3}\left(\frac{1}{a}\right) = -\frac{1}{abcd}$

Solution:

$$f(x)=\frac{1}{x}\Rightarrow f(a)=\frac{1}{a}, f(b)=\frac{1}{b}.$$

$$f(a,b)=\underset{b}{\Delta}\left(\frac{1}{a}\right)=\frac{f(b)-f(a)}{b-a}=\frac{\frac{1}{b}-\frac{1}{a}}{b-a}=\frac{\frac{a-b}{ba}}{b-a}=-\frac{1}{ab}$$

If $$f(a,b,c)=\frac{f(b,c)-f(a,b)}{c-a}=\frac{-\frac{1}{bc}+\frac{1}{ab}}{c-a}=\frac{\frac{-ab+bc}{abc}}{c-a}=\frac{1}{abc}$$

$$f(a,b,c,d)=\frac{f(b,c,d)-f(a,b,c)}{d-a}=\frac{\frac{1}{bcd}-\frac{1}{abc}}{d-a}=-\frac{1}{abcd}$$

$$\therefore \underset{bcd}{\Delta^3}\left(\frac{1}{a}\right)=-\frac{1}{abcd}$$

7. Find the divided difference for the the following data:

x	2	5	10
y	5	29	109

Solution:

x	y=f(x)	$\Delta f(x)$	$\Delta^2 f(x)$
2	5		
		$\frac{29-5}{5-2}=8$	
5	29		$\frac{16-8}{10-2}=1$
		$\frac{109-29}{10-5}=16$	
10	109		

8. Give the Newton,s divided difeerence interpolation formula.

Solution:

$$f(x)=f(x_0)+(x-x_0)f(x_0,x_1)+(x-x_0)(x-x_1)f(x_0,x_1,x_2)+\ldots\ldots\ldots\ldots$$
$$+(x-x_0)(x-x_1)\ldots\ldots\ldots\ldots(x-x_{n-1})f(x_0,x_1,x_2,\ldots\ldots,x_n)$$

9. State any two properties of divided differences.

Solution:

1) The divided differences are symmetrical in all their arguments.i.e, the value of any difference is independent of the order of the arguments.

2) The divided differences of the sum or difference of two functions is equal to the sum or difference of the corresponding separate divided difference.

10.From the divided difference for the the following data:

x	5	15	22
y	7	36	160

Solution:

x	y=f(x)	$\Delta f(x)$	$\Delta^2 f(x)$
5	7		
		$\frac{36-7}{15-5}=2.9$	
			$\frac{17.7-2.9}{22-5}=\frac{14.8}{17}$
15	36		
		$\frac{160-36}{22-15}=17.7$	
22	160		

11. Write the formula for cubic spline.

Solution:

$$s(x)=y(x)=\frac{1}{6h}\left[(x_i-x)^3M_{i-1}+(x-x_{i-1})^3M_i\right]+\frac{1}{h}(x_i-x)\left[y_{i-1}-\frac{h^2}{6}M_{i-1}\right]+$$

$$\frac{1}{h}(x-x_{i-1})\left[y_i-\frac{h^2}{6}M_i\right]$$

The recurrence relation for the spline is

$$M_{i-1}+4M_i+M_{i+1}=\frac{6}{h^2}\left[y_{i-1}-2y_i+y_{i+1}\right] \quad \text{for} \quad i=1,2,\ldots(n-1)$$

12. Derive Newton's backward difference formula by using operator method.[Gregory Newton's backward difference interpolation formula].

Solution:

$$y(x)=f(x)=P_n(x)$$

$$=y_n+\frac{u}{1!}\nabla y_n+\frac{u(u+1)}{2!}\nabla^2 y_n+\frac{u(u+1)(u+2)}{3!}\nabla^3 y_n+\ldots\ldots$$

$$+\frac{u(u+1)(u+2)\ldots..(u+(n-1))}{n!}\nabla^n y_n$$

$$\text{where} \quad u=\frac{x-x_n}{h}$$

13. State Gregory Newton's forward difference interpolation formula.

Solution:

$$y(x)=f(x)=P_n(x)$$

$$=y_0+\frac{u}{1!}\Delta y_0+\frac{u(u-1)}{2!}\Delta^2 y_0+\frac{u(u-1)(u-2)}{3!}\Delta^3 y_0+\ldots\ldots$$

$$+\frac{u(u-1)(u-2)\ldots..(u-(n-1))}{n!}\Delta^n y_0$$

$$\text{where} \quad u=\frac{x-x_0}{h}$$

14. Given f(0)=-2,f(1)=2 and f(2)=8.Find the polynomial using Newton's interpolation formula.

Solution:

x	y=f(x)	$\Delta f(x)$	$\Delta^2 f(x)$
0	-2	4	
1	2	6	2
2	8		

By Newton's forward difference interpolation formula

$$y(x) = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \dots\dots$$
$$+ \frac{u(u-1)(u-2).....(u-(n-1))}{n!}\Delta^n y_0$$

where $u = \frac{x - x_0}{h}$

Here

$x_0 = 0, h = 1 \Rightarrow u = x$

$$\therefore y(x) = -2 + \frac{x}{1!}(4) + \frac{x(x-1)}{2!}(2)$$
$$= -2 + 4x + x(x-1)$$
$$= x^2 + 3x - 2.$$

15.State the merits and demerits of Newton's forward and backward interpolation formula.

Merits:

Newton's forward and backward interpolation formula are applicable for interpolation near the beginning and end respectively of tabulated values.

Demerits:

Newton's forward and backward interpolation formula used only for equal intervals (or) equidistant intervals.

16.Using Newton's forward difference formula,write the formula for the first,second and third order derivatives at $x = x_0$

Solution:

$$\left(\frac{dy}{dx}\right)_{x=x_0} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \dots\dots\right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_0} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \frac{5}{6}\Delta^5 y_0 + \dots\dots\right]$$

$$\left(\frac{d^3y}{dx^3}\right)_{x=x_0} = \frac{1}{h^3}\left[\Delta^3 y_0 - \frac{3}{2}\Delta^4 y_0 + \dots\dots\right]$$

17. Construct an Newton difference table for the points (0,-1),(1,1),(2,1) and (3,-2).

Solution:

Difference table :

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	-1			
		1+1:2		
1	1		0-2:-2	
		1-1:0		-3+2:-1
2	1		-3-0:-3	
		-2-1:-3		
3	-2			

18. State Trapezoidal rule.

$$\int_{x_o}^{x_n} y\,dx = \frac{h}{2}\left[(y_0 + y_n) + 2(y_1 + y_2 + ... + y_{n-1})\right] \text{ where } h = \frac{b-a}{n}$$

$$= \frac{h}{2}[A + 2B]$$

where A=sum of the first and last ordinates & B=sum of the remaining ordinates.

19. Using Newton's backward difference formula,write the formula for the first,second and third order derivatives at $x = x_n$

Solution:

$$\left(\frac{dy}{dx}\right)_{x=x_n} = \frac{1}{h}\left[\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + \frac{1}{4}\nabla^4 y_n + ...\right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_n} = \frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \frac{5}{6}\nabla^5 y_n + ...\right]$$

$$\left(\frac{d^3y}{dx^3}\right)_{x=x_n} = \frac{1}{h^3}\left[\nabla^3 y_n + \frac{3}{2}\nabla^4 y_n + ...\right]$$

20. Find $\frac{dy}{dx}$ at x=1 from the following table:

x	1	2	3	4
y	1	8	27	64

Solution:

y	Δ	Δ^2	Δ^3
1			
	7		
8		12	
	19		6
27		18	
	37		
64			

$$\left(\frac{dy}{dx}\right)_{x=x_0} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0\right]$$

$$= (1)\left[7 - \frac{12}{2} + \frac{6}{3}\right] = 3$$

21.State Simpson’s 3/8 rule.

$$\int_{x_0}^{x_n} y\,dx = \frac{3h}{8}\left[(y_0 + y_n) + 3(y_1 + y_2 + y_4 + y_{5+} .... + y_{n-1}) + 2(y_3 + y_6 + ... + y_{n-3})\right] \text{ where } h = \frac{b-a}{n}$$

22.State Simpson’s 1/3 rule.

$$\int_{x_0}^{x_n} y\,dx = \frac{h}{3}\left[(y_0 + y_n) + 4(y_1 + y_3 + ... + y_{n-1}) + 2(y_2 + y_4 + ... + y_{n-2})\right] \text{ where } h = \frac{b-a}{n}$$

$$= \frac{h}{3}[A + 4B + 2C]$$

where A=sum of the first and last ordinates.
B=sum of the odd ordinates
C= sum of the even ordinates

23.Evaluate $\int_{1/2}^{1} \frac{1}{x}dx$ by Trapezoidal rule,dividing the range into 4 equal parts.

Solution:

$$h = \frac{1-\frac{1}{2}}{4} = \frac{1}{8}$$

x	1/2	5/8	6/8	7/8	8/8
f(x)	8/4	8/5	8/6	8/7	8/8

$$\therefore \int_{1/2}^{1} \frac{1}{x}dx = \frac{1}{8}.\frac{1}{2}\left[\left(\frac{8}{4}+\frac{8}{8}\right)+2\left(\frac{8}{5}+\frac{8}{6}+\frac{8}{7}\right)\right]$$

$$= \frac{1}{16}\left[3+2\left(\frac{856}{210}\right)\right] = \frac{1171}{1680} = 0.6971$$

24.Evaluate $\int_0^1 \frac{dx}{1+x^2}$ with h=1/6 by Trapezoidal rule.

Solution:

Given $f(x) = \int_0^1 \frac{dx}{1+x^2}, h = 1/6$

x	0	1/6	2/6	3/6	4/6	5/6	6/6
f(x)	1	36/37	9/10	4/5	9/13	36/31	1/2

$$\therefore \int_0^1 \frac{dx}{1+x^2} = \frac{1}{12}\left[\left(1+\frac{1}{2}\right)+2\left(\frac{36}{37}+\frac{9}{10}+\frac{4}{5}+\frac{9}{13}+\frac{36}{31}\right)\right]$$

$$= 0.7842$$

25.Using Simpson,s rule(one-third) evaluate $\int_0^1 xe^x dx$ taking 4 intervals.Compare your result with actual value.

Solution:

Given $f(x) = xe^x; h = \frac{1}{4} = 0.25$

x	0	0.25	0.5	0.75	1
f(x)	1	0.321	0.824	1.588	2.718

By Simpson’s 1/3 rule

$$\therefore \int_0^1 xe^x dx = \frac{0.25}{3}\left[(0+2.718)+2(0.321+1.588)+4(0.524)\right]$$

$$= 0.819 = 1$$

By actual integration

$$\int_0^1 xe^x dx = [xe^x - e^x]_0^1 = e^1 - e^1 - (0-1) = 1$$

26.Calculate $\int_{0.5}^{0.7} e^{-x}\sqrt{x}dx$ taking 5 ordinates by Simpson’s 1/3 rule.

Solution:

Given $f(x) = e^{-x}\sqrt{x}; h = \frac{0.7-0.5}{4} = 0.05$

x	0.5	0.55	0.6	0.65	0.7
f(x)	0.4289	0.4279	0.4251	0.4209	0.4155

By Simpson's 1/3 rule

$$\therefore \int_0^1 e^{-x}\sqrt{x}dx = \frac{0.05}{3}\left[(0.4289+0.4155)+2(0.4279+0.4279)+4(0.4251)\right]$$
$$= 0.849$$

27. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by Simpson,s 3/8 rule &check by actual integration.

Solution:

Given $f(x) = \frac{1}{1+x^2}; h = \frac{6-0}{6} = 1$

x	0	1	2	3	4	5	6
f(x)	1	0.500	0.200	0.100	0.0588	0.0384	0.2702

By Simpson's 3/8 rule,

$$\therefore \int_0^6 \frac{dx}{1+x^2} = \frac{3h}{8}\left[(y_0+y_n)+3(y_1+y_2+.....+y_{n-1})+2(y_3+y_6+...+y_{n-3})\right]$$
$$= \frac{3}{8}\left[(1+0.2702)+3(0.5+0.2+0.0588+0.0384)+2(0.1)\right]$$
$$= 1.3570$$

By actual integration,

$$\int_0^6 \frac{dx}{1+x^2} = \left[\tan^{-1}(x)\right]_0^6 = \tan^{-1}(6) = 1.4056$$

28. What are the errors in Trapezoidal & Simpson's rules of numerical integration?

Solution:

Error in Trapezoida rule $|E| < \frac{(b-a)}{12}h^2M$; order is h^2

Error in Simpson's rule $|E| < \frac{(b-a)}{180}h^4M$; order is h^4

29. In order to evaluate $\int_{x_0}^{x_n} y\,dx$ by Simpson's rule,what is the restriction on the number of intervals.

Solution:

Simpson's 1/3 rule:The number of ordinates is odd (0r) the interval number is even.

Simpson's 3/8 rule:The interval is a multiple of 3.

30. Using Simpson's rule find $\int_0^4 e^x dx$, given $e^0 = 1, e^1 = 2.72, e^2 = 7.39, e^3 = 20.09$ and $e^4 = 54.6$

Solution:

$y = e^x; h = 1$

x	0	1	2	3	4
y	1	2.72	7.39	20.09	54.6

By Simpson's rule,

$$\therefore \int_0^4 e^x dx = \frac{1}{3}\left[(1+54.6)+2(7.39)+4(2.72+20.09)\right] = 53.87$$

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DEPARTMENT OF MATHEMATICS

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PART B

1. Use Lagrange's interpolation, calculate the profit in the year 2000 from the following data

Year :	1997	1999	2001	2002
Profit in lakhs of RS :	43	65	159	248

2 Find the third degree polynomial of f(x) satisfying the following data

X :	1	3	5	7
Y :	24	120	336	720

3 Find the polynomial f(x) by using Lagrange's formula and hence find f(3) for

X :	0	1	2	5
f(x) :	2	3	12	147

4 Using Lagrange's interpolation formula find y(10) given that y(5)=12,y(6)=13,y(9)=14,y(11)=16

5 Obtain the root of f(x) =0 by Lagrange's inverse interpolation given that f(30) = -30, f(34) = -3, f(38) = 3, f(42) = 18.

6 Find the missing term in the following table using Lagrange's interpolation

X :	0	1	2	3	4
Y:	1	3	9	-	81

7 Using Newton's divided difference formula, find u(3)given u(1) = - 26, u(2) = 12, u(4) = 256, u(6) = 844.

8 Find f(x) as a polynomial in x for the following data by newton's divided difference formula:

X :	-4	-1	0	2	5
f(x):	1245	33	5	9	1335

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9 Find f(8) by newton's divided difference formulae for the data:

X	4	5	7	10	11	13
f(x)	48	100	294	900	1210	2028

10 Find f'(3) and f''(3) for one following data:

X	3.0	3.2	3.4	3.6	3.8	4.0
F(X)	-14	-10.032	-5.296	-0.256	6.672	14

11 Compute f'(0) and f''(4) from the data

X:	0	1	2	3	4
Y:	1	2.718	7.381	20.086	54.598

12 The following data gives the velocity of a particle for 20 seconds at an interval of 5 seconds .find the initial acceleration using the entire data

Time(sec)	0	5	10	15	20
Velocity(m/sec)	0	3	14	69	228

13 Find the maximum and minimum value of y tabulated below:

X:	-2	-2	0	1	2	3	4
Y:	2	-.25	0	-.25	2	15.75	56

14 Using trapezoidal rule ,evaluate $\int_{-1}^{1} \frac{dx}{1+x^2}$ taking 8 intervals

15 Evaluate $\int_{0}^{1} \frac{dx}{1+x^2}$ with h=$\frac{1}{6}$ by trapezoidal rule.

16 Evaluate the integral $\int_{1}^{2} \frac{dx}{1+x^3}$ by using trapezoidal rule with two sub intervals.

17 Dividing the range into 10 equal parts, find the value of $\int_{0}^{\pi/2} sinx\, dx$ by (i) Trapezoidal rule (ii) Simpson's rule.

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18 Using Simpson's one third rule evaluate $\int_0^1 xe^x\,dx$ taking 4 intervals. Compare your result with actual value.

19 Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using (i) Trapezoidal rule (ii) Simpson's rule. Also check up result by actual integration.

20 By dividing the range into ten equal parts, Evaluate $\int_0^{\pi} sinx\,dx$ by using (i) Trapezoidal rule (ii) Simpson's rule. Also check up result by actual integration.