SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore– 35

DEPARTMENT OF MATHEMATICS

UNIT–V NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONS

PART A

1. State the disadvantages of Taylor's method.

Sol. In the differential equation dy/dx = f(x,y) , the f(x,y) function may have a complicated algebrical structure . Then the evaluation of higher order derivatives may become tedious.

2. Write down the fourth order Taylor's Algorithm.

Sol. $\mathbf{y_{m+1} = y_m + hy_m^1 + \left(\frac{h^2}{2!}\right)y_m^{11} + \left(\frac{h^3}{3!}\right)y_m^{111} + ..}$

3. Solve $y' = x + y; \quad y(0) = 1$ by Taylor's series method. Find the values y at x = 0.1.

Solution:

$$\text{Given } y' = x + y; x_0 = 0, y_0 = 1, h = 0.1$$

$$y' = x + y \Rightarrow y_0' = x_0 + y_0 = 0 + 1 = 1$$

$$y'' = 1 + y' \Rightarrow y_0'' = 1 + y_0' = 1 + 1 = 2$$

$$y''' = y'' \Rightarrow y_0''' = y'' = 2$$

$$y^{iv} = y''' \Rightarrow y_0^{iv} = y''' = 2$$

$$y_1 = y_0 + \frac{h}{1!}y_0' + \frac{h^2}{2!}y_0'' + \frac{h^3}{3!}y_0''' + ...$$

we know that $y(0.1) = 1 + 0.1 + 0.01 + 0.0003 +$

$= 1.11033$

4. Write the Euler algorithm to find the differential equation $\frac{dy}{dx} = f(x,y)$.

Solution:

$y_{n+1} = y_n + hf(x_n, y_n)$ for the interval (x_n, y_n) when n=0,1,2,…

5. State modified Euler's algorithm to solve $y' = f(x,y), y(x_0) = y_0 \quad \text{at} \quad x = x_0 + h$

Solution:

$$y_{n+1} = y_n + hf\left[x_n + \frac{h}{2}, y_n + \frac{h}{2}f(x_n, y_n)\right] \quad \text{when } n=0,1,2,\ldots$$

6. Using Euler's method solve $y' = x + y + xy , y(0) = 1.$ Compute y at x=0.1 by taking h=0.05.

Solution:

$$\begin{aligned}
&\text{Given}: f(x,y) = x + y + xy \\
&x_0 = 0, y_0 = 1; h = 0.05 \\
&y_1 = y_0 + hf(x_0, y_0) \\
&= 1 + 0.05[x_0 + y_0 + x_0 y_0] \\
&= 1 + 0.05[0 + 1 + 0] \\
&= 1.05
\end{aligned}$$

7. Compute y at x = 0.25 by modified Euler method given $y' = 2xy, y(0) = 1.$

Solution:

$$\begin{aligned}
&\text{Given} \quad y' = 2xy \\
&x_0 = 0; y_0 = 1; h = 0.25 \\
&y_1 = y_0 + hf\left[x_0 + \frac{h}{2}, y_0 + \frac{h}{2} f(x_0, y_0)\right] \\
&= 1 + 0.25f\left[0 + \frac{0.25}{2}, 1 + \frac{0.25}{2}(2x_0 y_0)\right] \\
&= 1 + 0.25f[0.25, 1] \\
&= 1 + 0.25[2(0.125)(1)] \\
&= 1 + 0.625 = 1.0625
\end{aligned}$$

8. Write down the Runge- Kutta method formula of second order to solve $y' = f(x,y)$ with $y(x_0) = y_0$.

Solution:

$$\begin{aligned}
&k_1 = hf(x,y) \\
&k_2 = hf\left[x + \frac{h}{2}, y + \frac{k_1}{2}\right] \\
&\text{and} \quad \Delta y = k_2 \\
&y_1 = y_0 + \Delta y
\end{aligned}$$

9. Write down the Runge kutta method formula of fourth order to solve $\frac{dy}{dx} = f(x,y)$ with $y(x_0) = y_0$.

Solution:

$$k_1 = hf(x,y)$$
$$k_2 = hf\left[x+\frac{h}{2}, y+\frac{k_1}{2}\right]$$
$$k_3 = hf\left[x+\frac{h}{2}, y+\frac{k_2}{2}\right]$$
$$k_4 = hf\left[x+h, y+k_3\right]$$
and $\Delta y = \frac{1}{6}[k_1+2k_2+2k_3+k_4]$
$$y_1 = y_0 + \Delta y$$

10. Compare Taylor's series and RK method.

Solution:

R.K methods do not require prior calculation of higher derivatives of y(x) as the Taylor method does. Also the RK formulas involve the computation of f(x,y) at various position,instead of derivatives and this function occurs in the given equation.

11. Write Milne's predictor corrector formula.

Solution:

Milne's predictor formula

$$y_{n+1,p} = y_{n-3} + \frac{4h}{3}\left[2y'_{n-2} - y'_{n-1} + 2y'_n\right]$$

Milne's corrector formula

$$y_{n+1,c} = y_{n-1} + \frac{h}{3}\left[y'_{n-1} + y'_n + 2y'_{n+1}\right]$$

12. How many prior values are required to predict the next value in Milne's method?

Solution:

Four prior values

$$y(x_0)=y_0;\ y(x_1)=y_1;\ y(x_2)=y_2;\ y(x_3)=y_3$$

13. What is the error term in Milne's corrector formula?

Solution:

The error term is $-\frac{h}{90}\Delta^4 y'_0$

14. What is the error term in Milne's predictor formula?

Solution:

The error term is $\frac{14h}{45}\Delta^4 y_0'$

15. Write down Adams Bashforth predictor corrector formula.

Solution:

Adams Bashforth predictor formula is

$$y_{n+1,p} = y_n + \frac{h}{24}\left[55y_n' - 59y_{n-1}' + 37y_{n-2}' - 9y_{n-3}'\right]$$

Adams Bashforth corrector formula is

$$y_{n+1,c} = y_n + \frac{h}{24}\left[9y_{n+1}' + 19y_n' - 5y_{n-1}' + y_{n-2}'\right]$$

16. How many prior values are required to predict the next value in Adam's method?

Solution:

Four prior values.

17. What is the predictor –corrector method of solving a differential equation?

Solution:

Predictor –Corrector methods are methods which require the values of y at $x_n, x_{n-1}, x_{n-2}, ...$ for computing the value of y at x_{n+1}. We first use a formula to find the value of y at x_{n+1} and this is known as a predictor formula. The value of y so got is improved or corrected by another formula known as corrector formula.

18. Compare Runge-Kutta method and Predictor-Corrector method.

Solution:

Runge-Kutta method:

RK methods are self-starting ,since they do not use information from previously calculated points.

Rk method is single step method.In these methods,it is not possible to get any information about truncation.

Predictor-Corrector method:

Predictor-Corrector method is not self starting,since these methods require information about four prior points.

It is multi step method.In these methods,it is possible to get easily a good estimate of truncation error.

19. Write the merits and demerits of the Taylor's method of solution

Sol. The method gives a straight forward adaption of classic calculus to develop the solution as an infinite series. It is a powerful single step method if we are able to find the successive derivatives easily. If f(x,y) involves some complicated algebraic structures then the calculation of higher derivatives become tedious and the method fails.

20. Which is better Taylor's method or R.K. Method ?
Sol. R.K methods do not require prior calculation of higher derivatives of y(x) , as the Taylor's method does . Since the differential equations using in applications are often complicated, the calculation of derivatives may be difficult.

UNIT–V NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONS

PART -B QUESTIONS

1. By Taylor's series method find y(0.1) given that y^{11} = y+ xy ; y(0) = 1; $y^1(0)$= 0.

2. Solve the system of equations dy/dx =z –x^2 , dz/dx =y + x with y(0) =1, z(0) =1 by taking h=0.1 , to get y(0.1) and z(0.1) . Here y and z are dependent variable and x is independent.

3. Using Euler's method find y(0.2) and y(0.4) from dy/dx =x+ y , y(0)= 1, with h=0.2.

4. Using modified Euler's method compute y(0.1) with h= 0.1 from y^1 = y – (2x/y), y(0)=1.

5. Given dy/dx= x^3 + y , y(0)=2 . Compute y(0.2) , y(0.4) ,y(0.6) by Runge-Kutta method of Fourth order.

6. Solving the system of differential equation dy/dx = xz + 1; dz/dx = -xy for x = 0.3 using fourth order Runge- Kutta method , the initial values are x=0 , y=0 , z= 1.

7. Determine the value of y(0.4) using Milne's method given y^1= xy +y^2, y(0)=1 ; Use Taylor series to get the values of y(0.1) ,y(0.2) , y(0.3).

8. Using Runge-Kutta method calculate y(0.1), y(0.2), y(0.3) given that $\frac{dy}{dx}-\frac{2xy}{1+x^2}=1$, y(0)= 0. Taking these values as starting values find y(0.4) by milne's method.

9. Find y(0.1) , y(0.2) , y(0.3) from $\frac{dy}{dx}=xy+y^2$, y(0) = 1 by using R.K method and hence obtain y(0.4) using Adam's method.