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## Department of Biomedical Engineering

**Course Name: 19BMB304 & Biomedical Image Processing**

**III Year : VI Semester**

**Unit III –IMAGE RESTORATION AND SEGMENTATION**

**Topic : Notch Filters – Optimum Notch Filtering**

# Image Restoration

- Image restoration vs. image enhancement
  - Enhancement:
    - largely a subjective process
    - Priori knowledge about the degradation is not a must (sometimes no degradation is involved)
    - Procedures are heuristic and take advantage of the psychophysical aspects of human visual system
  - Restoration:
    - more an objective process
    - Images are degraded
    - Tries to recover the images by using the knowledge about the degradation

# Image Restoration

Image restoration concerns the removal or reduction of degradations which have occurred during the acquisition of the image. Such degradations may include noise, which are errors in the pixel values, or optical effects such as out of focus blurring, or blurring due to camera motion. We shall see that some restoration techniques can be performed very successfully using neighbourhood operations, while others require the use of frequency domain processes. Image restoration remains one of the most important areas of image processing, but in this chapter the emphasis will be on the techniques for dealing with restoration, rather than with the degradations themselves, or the properties of electronic equipment which give rise to image degradation.

# An Image Degradation Model

- Two types of degradation
  - Additive noise
    - Spatial domain restoration (denoising) techniques are preferred
  - Image blur
    - Frequency domain methods are preferred
- We model the degradation process by a degradation function h(x,y), an additive noise term, $\eta(x,y)$, as $g(x,y)=h(x,y)*f(x,y)+\eta(x,y)$
  - f(x,y) is the (input) image free from any degradation
  - g(x,y) is the degraded image
  - $*$ is the convolution operator
  - The goal is to obtain an estimate of f(x,y) according to the knowledge about the degradation function h and the additive noise $\eta$
  - In frequency domain: $G(u,v)=H(u,v)F(u,v)+N(u,v)$
- Three cases are considered in this Chapter
  - $g(x,y)=f(x,y)+\eta(x,y)$ (5-2~5-4)
  - $g(x,y)=h(x,y)*f(x,y)$ (5-5~5-6)
  - $g(x,y)=h(x,y)*f(x,y)+\eta(x,y)$ (5-7~5-9)

# A Model of the Image Degradation/Restoration Process

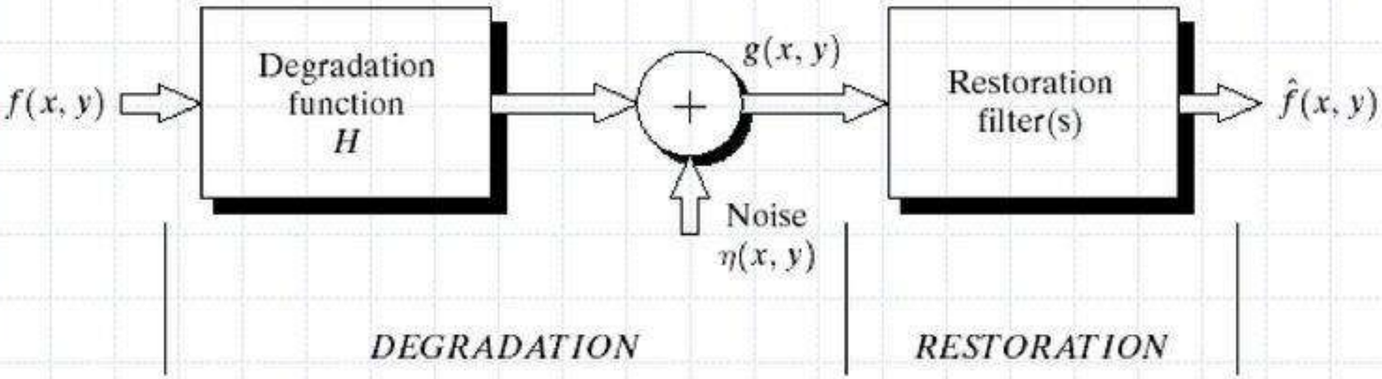



**FIGURE 5.1** A model of the image degradation/restoration process.

# A Model of the Image Degradation/Restoration Process

In the spatial domain, we might have an image $f(x, y)$, and a spatial filter $h(x, y)$ for which convolution with the image results in some form of degradation. For example, if $h(x, y)$ consists of a single line of ones, the result of the convolution will be a motion blur in the direction of the line. Thus we may write

$$g(x, y) = f(x, y) * h(x, y)$$

for the degraded image, where the symbol $*$ represents spatial filtering. However, this is not all. We must consider noise, which can be modelled as an additive function to the convolution. Thus if $n(x, y)$ represents random errors which may occur, we have as our degraded image:

$$g(x, y) = f(x, y) * h(x, y) + n(x, y).$$

# Periodic Noise Reduction by Frequency Domain Filtering

- Lowpass and highpass filters for image enhancement have been studied
- Bandreject, bandpass, and notch filters as tools for periodic noise reduction or removal are to be studied in this section.

# Notch Filters

Notch filters that pass, rather than suppress:

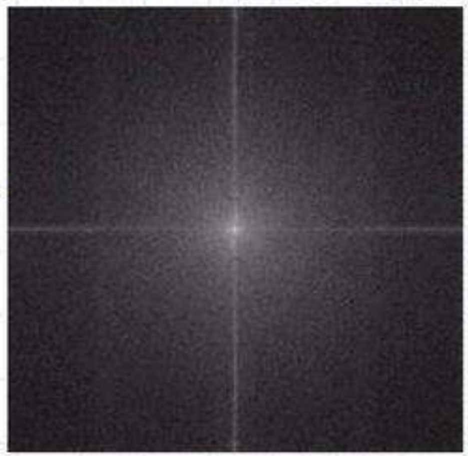
$$H_{np}(u,v) = 1 - H_{nr}(u,v)$$

- *NR* filters become highpass filters if $u_0 = v_0 = 0$
- *NP* filters become lowpass filters if $u_0 = v_0 = 0$

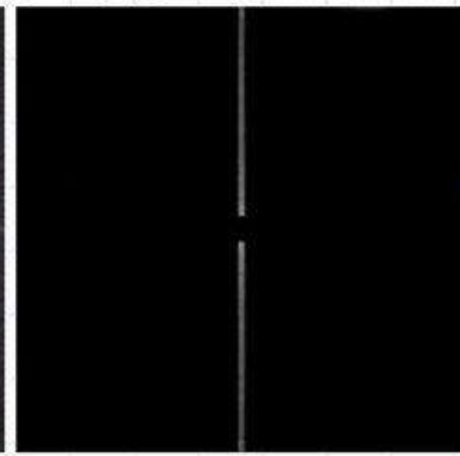
# Notch Filters

You can see the effect of scan lines

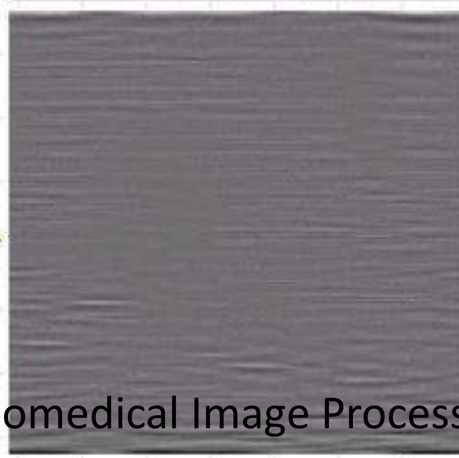
Spectrum of image



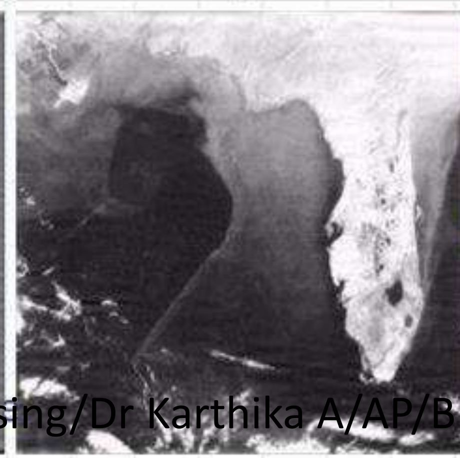
Notch pass filter



IFT of NP filtered image



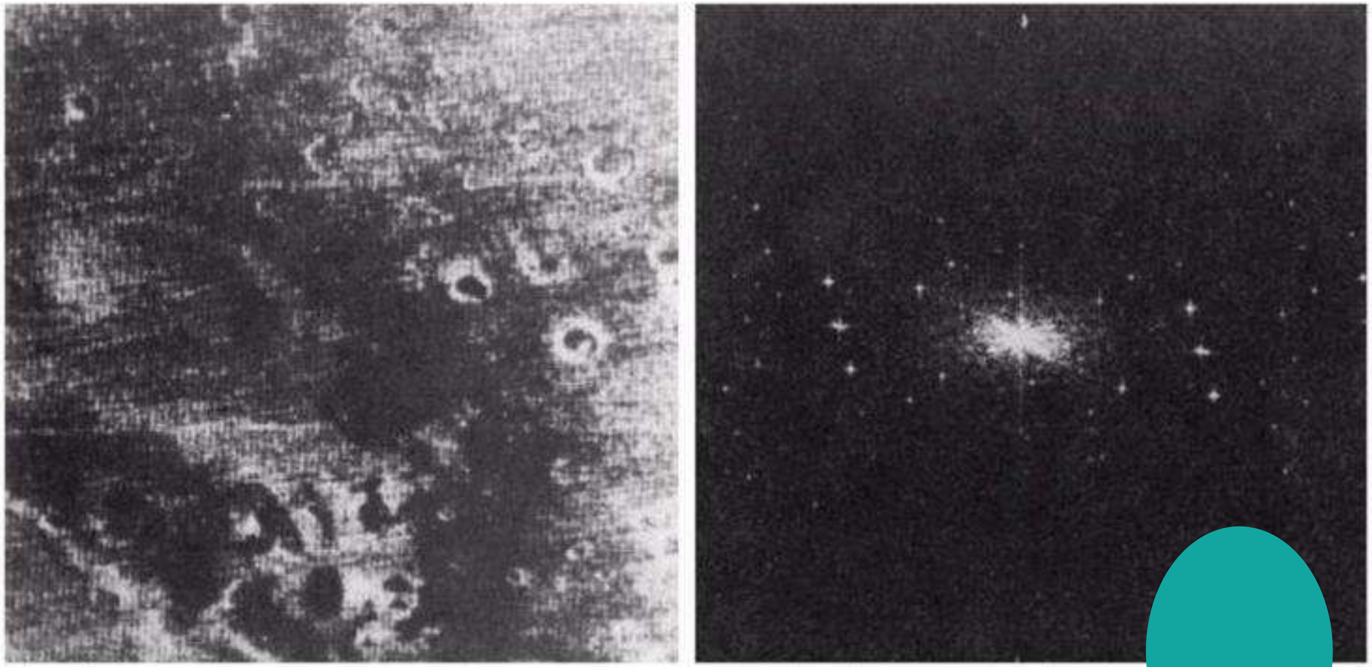
Result of NR filter



# Optimum Notch Filtering

a b

**FIGURE 5.20**
(a) Image of the Martian terrain taken by *Mariner 6*.
(b) Fourier spectrum showing periodic interference.
(Courtesy of NASA.)



# Optimum Notch Filtering

- In the ideal case, the original image can be restored if the noise can be estimated completely.
  - That is: $f(x, y) = g(x, y) - \eta(x, y)$

- However, the noise can be only partially estimated. This means the restored image is not exact.
  - Which means $\hat{f}(x, y) = g(x, y) - \hat{\eta}(x, y)$

$$\hat{\eta}(x, y) = IFT\{H(u,v)G(u,v)\}$$

# Optimum Notch Filtering

- In this section, we try to improve the restored image by introducing a modulation function
  - $\hat{f}(x,y) = g(x,y) - w(x,y)\hat{\eta}(x,y)$
  - Here the modulation function is a constant within a neighborhood of size (2a+1) by (2b+1) about a point (x,y)
  - We optimize its performance by minimizing the local variance of the restored image at the position (x,y)

$$\sigma^2(x,y) = \frac{1}{(2a+1)(2b+1)} \sum_{s=-a}^{a} \sum_{t=-b}^{b} \left[ \hat{f}(x+s,y+t) - \bar{\hat{f}}(x,y) \right]^2$$

$$\bar{\hat{f}}(x,y) = \frac{1}{(2a+1)(2b+1)} \sum_{s=-a}^{a} \sum_{t=-b}^{b} \hat{f}(x+s,y+t)$$

# Optimum Notch Filtering

Points on or near Edge of the image can be treated by considering partial neighborhoods

$$\sigma^2(x,y) = \frac{1}{(2a+1)(2b+1)} \sum_{s=-a}^{a} \sum_{t=-b}^{b} \{[g(x+s,y+t) - w(x+s,y+t)\hat{\eta}(x+s,y+t)] - [\bar{g}(x,y) - \overline{w(x,y)\hat{\eta}(x,y)}]\}^2$$

Assumption: $w(x+s,y+t) = w(x,y) \quad for \quad -a \le s \le a \; and \; -b \le t \le b$

$$\Rightarrow \overline{w(x,y)\hat{\eta}(x,y)} = w(x,y)\bar{\hat{\eta}}(x,y)$$

# Optimum Notch Filtering

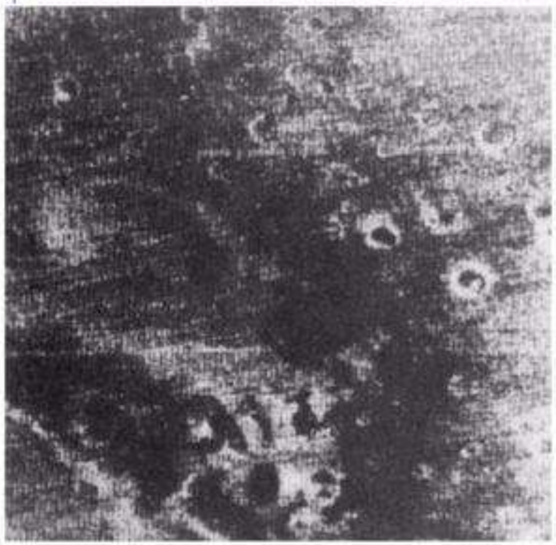
$$\sigma^2(x,y) = \frac{1}{(2a+1)(2b+1)} \sum_{s=-a}^{a} \sum_{t=-b}^{b} \{[g(x+s,y+t) - w(x+s,y+t)\hat{\eta}(x+s,y+t)] - [\bar{g}(x,y) - w(x,y)\overline{\hat{\eta}}(x,y)\}^2$$

To minimize $\sigma^2(x,y)$

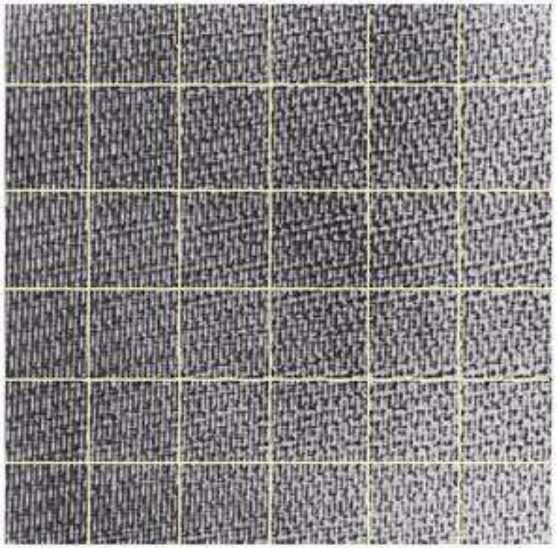
$$\frac{\partial \sigma^2(x,y)}{\partial w(x,y)} = 0$$

$$\Rightarrow w(x,y) = \frac{\overline{g(x,y)\hat{\eta}(x,y)} - \bar{g}(x,y)\overline{\hat{\eta}}(x,y)}{\overline{\hat{\eta}^2}(x,y) - \overline{\hat{\eta}}^2(x,y)}$$

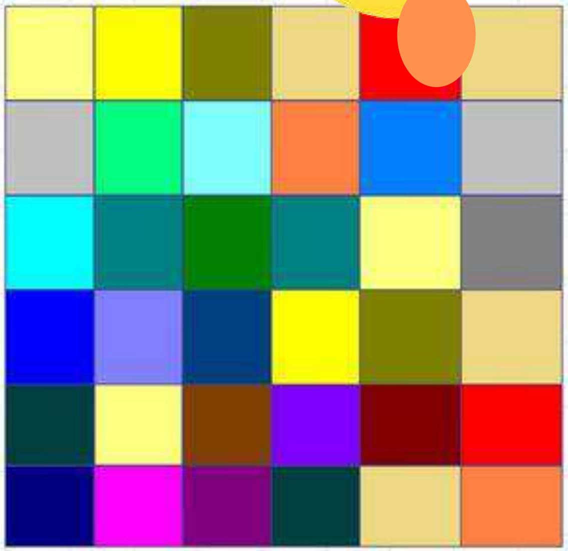
# Optimum Notch Filtering



$g(x, y)$



$\hat{\eta}(x, y)$



$w(x, y)$

$$\hat{f}(x, y) = g(x, y) - w(x, y)\hat{\eta}(x, y)$$

# Optimum Notch Filtering

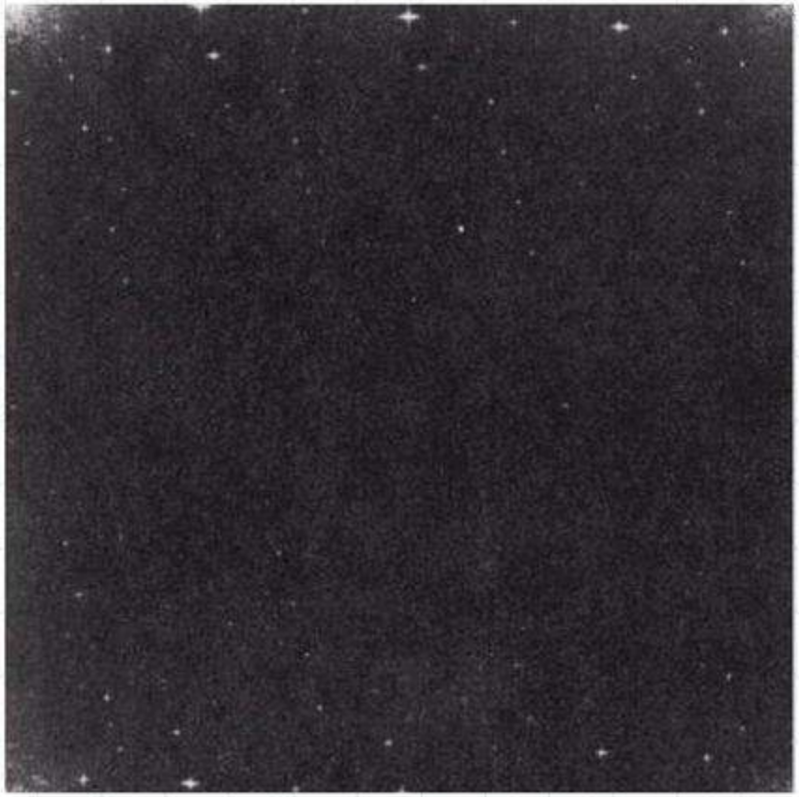
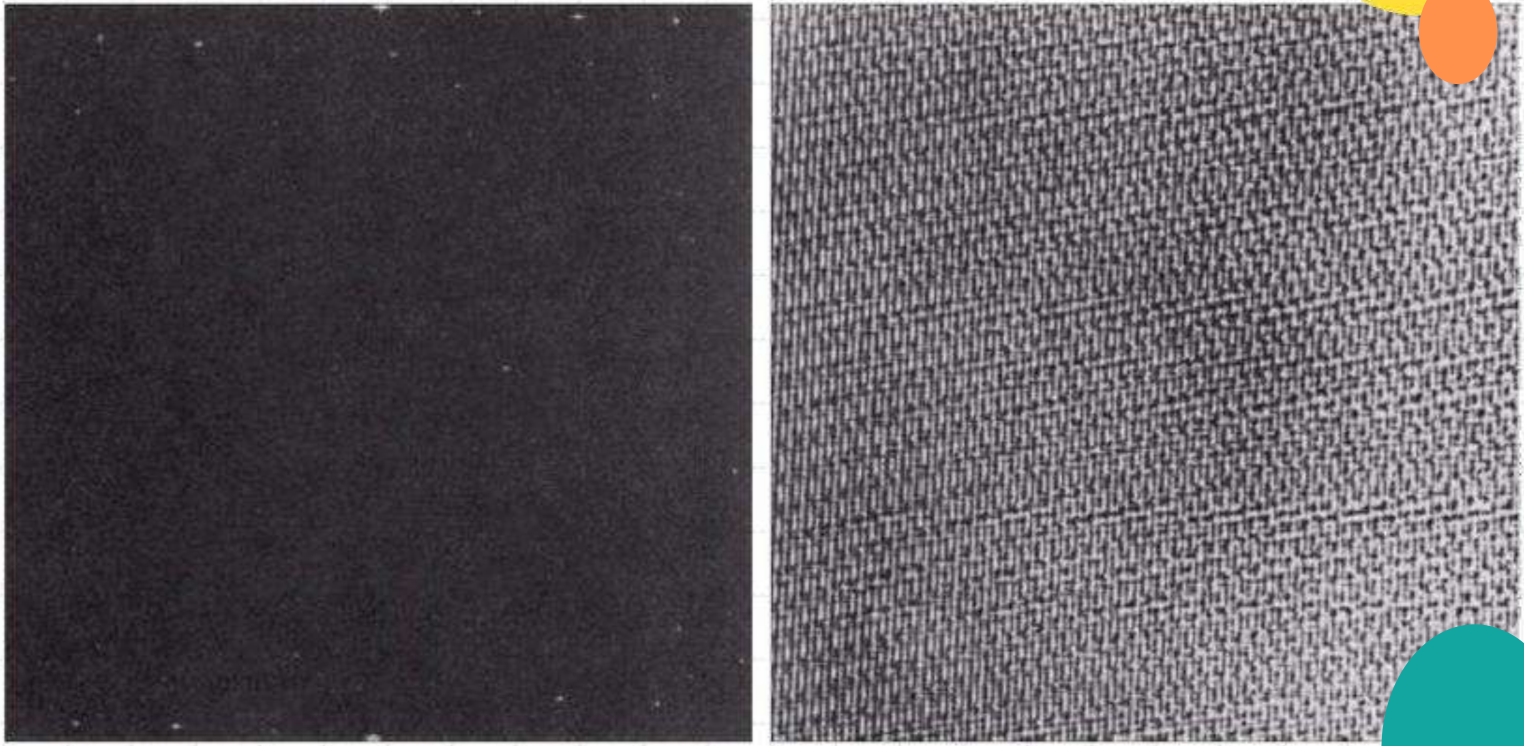


**FIGURE 5.21** Fourier spectrum (without shifting) of the image shown in Fig. 5.20(a). (Courtesy of NASA.)

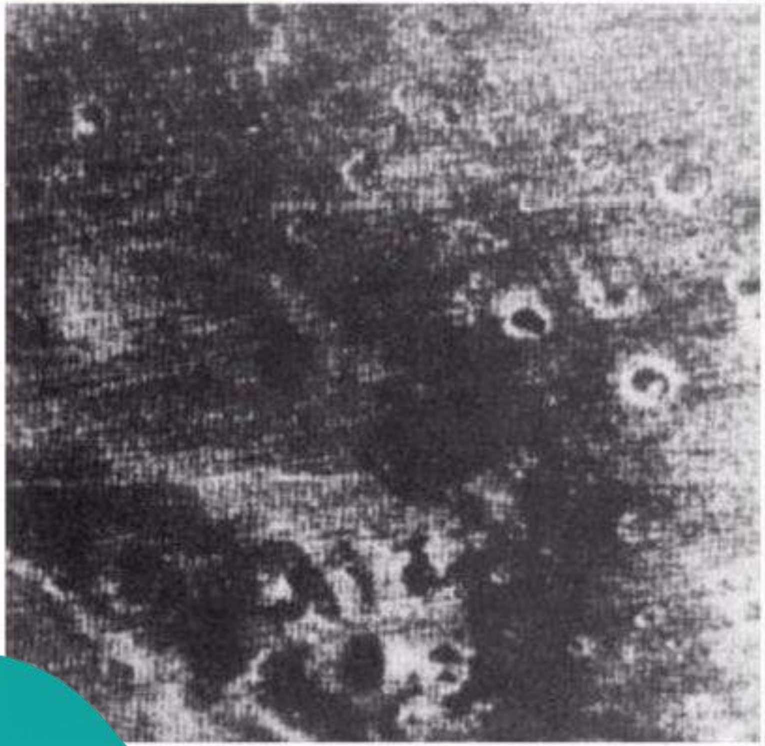
# Optimum Notch Filtering



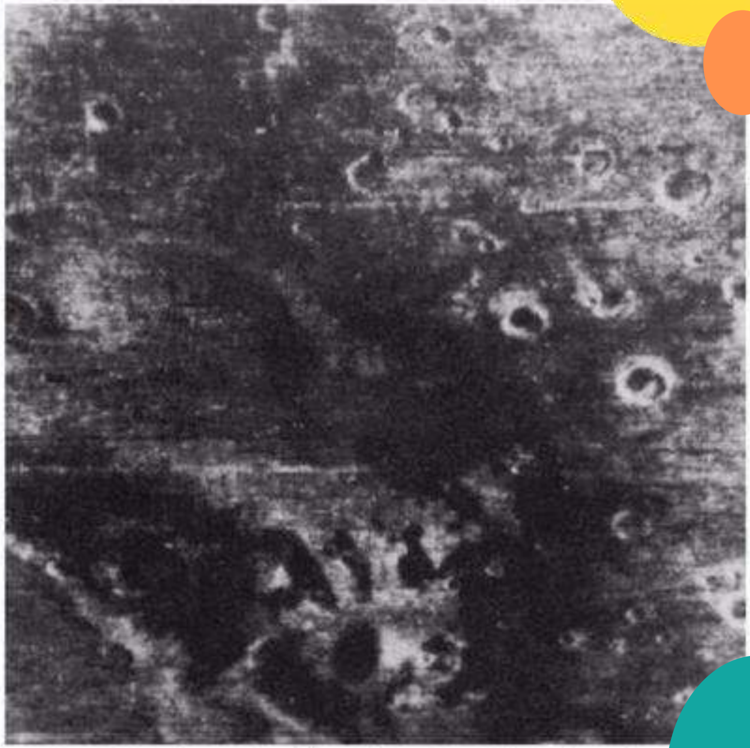
a b

**FIGURE 5.22** (a) Fourier spectrum of $N(u, v)$, and (b) corresponding noise interference pattern $\eta(x, y)$. (Courtesy of NASA.)

# Optimum Notch Filtering



$g(x, y)$



$\hat{f}(x, y)$

# Thank You