SNS COLLEGE OF TECHNOLOGY

(AN AUTONOMOUS INSTITUTION)

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Accredited by NBA & Accredited by NAAC with 'A++' Grade,
Recognized by UGC saravanampatti (post), Coimbatore-641035.

Department of Biomedical Engineering

Course Name: 19BMB304 & Biomedical Image Processing

III Year : VI Semester

Unit III –IMAGE RESTORATION AND SEGMENTATION

Topic : Inverse Filtering

Image Restoration Techniques

- Inverse of degradation process
- Depending on the knowledge of degradation, it can be classified into

Deterministic	Random
If prior knowledge about degradation is known	If not known

Linear	Non-linear
Restore the image by a filter e.g. Inverse Filtering Drawback: ringing artifacts near edges	Nonlinear function is used Ringing artifacts is reduced

Image Restoration Model

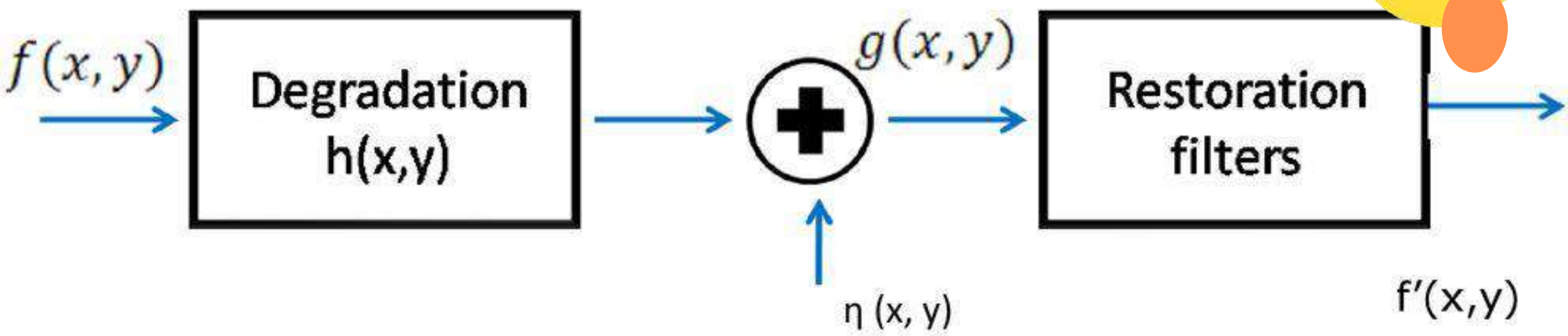


Image Restoration Model

- In this model,
  - f(x,y) → input image
  - h(x,y) → degradation
  - f'(x,y) → restored image
  - η(x,y) → additive noise
  - g(x,y) → degraded image

Image Restoration Model

- In spatial domain,

$$g(x,y) = f(x,y) * h(x,y)$$

and in frequency domain

$$G(k,l) = F(k,l).\ H(k,l)$$

Where G,F and H are fourier transform of g,f and h

Linear Restoration Technique

- They are quick and simple
- But limited capabilities
- It includes
  - Inverse Filter
  - Pseudo Inverse Filter
  - Wiener Filter
  - Constrained Least Square Filter

Inverse Filtering

- If we know exact PSF and ignore noise effect, this approach can be used.
- In practice PSF is unknown and degradation is affected by noise and hence this approach is not perfect.
- Advantage - Simple

Inverse Filtering

- From image restoration model

$$g(m, n) = f(m, n) * h(m, n) + \eta(m, n)$$

- For simplicity, the co-ordinate of the image are ignored so that the above equation becomes $g = Hf + \eta$
- Then the error function becomes $\eta = g - Hf$

Inverse Filtering

- We wish to ignore η and use $\hat{f}$ to approximate under least square sense. Then the error function is given as

$$J(\hat{f}) = \|g - H\hat{f}\|^2 = g^2 + H^2\hat{f}^2 - 2gH\hat{f}$$

- To find the minimum of $J(\hat{f})$, the above equation is differentiated wrt $\hat{f}$ and equating it to zero

$$\frac{\partial J(\hat{f})}{\partial \hat{f}} = 0 + 2H^2\hat{f} - 2gH = -2H^T(g - H\hat{f})$$

Inverse Filtering

- Solving for $\hat{f}$, we get

$$\hat{f} = \left(H^T H\right)^{-1} H^T g = H^{-1} g$$

- Taking fourier transform on both sides we get

$$\hat{F}(k,l) = \frac{G(k,l)}{H(k,l)}$$

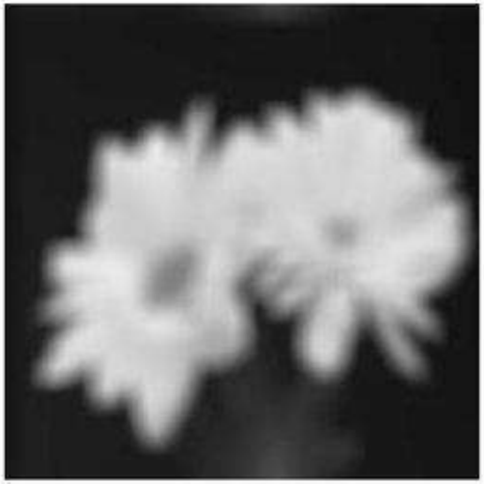
- The restored image in spatial domain is obtained by taking Inverse Fourier Transform as

$$\hat{f}(m,n) = \Im^{-1}\left[\hat{F}(k,l)\right] = \Im^{-1}\left[\frac{G(k,l)}{H(k,l)}\right]$$

Inverse Filtering

Original Image

Degraded Image



Restored Image

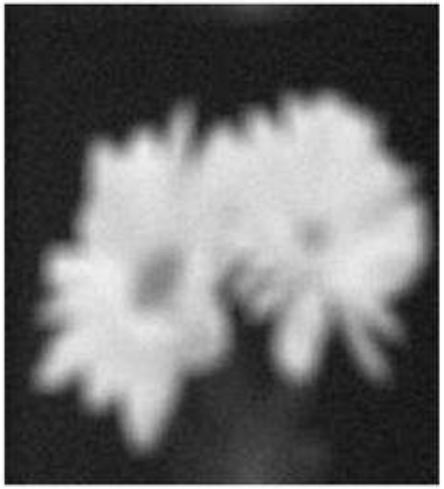
Inverse Filtering

- Advantages:
  - It requires only blur PSF
  - It gives perfect reconstruction in the absence of noise
- Drawbacks:
  - It is not always possible to obtain an inverse (singular matrices)
  - If noise is present, inverse filter amplifies noise. (better option is wiener filter)

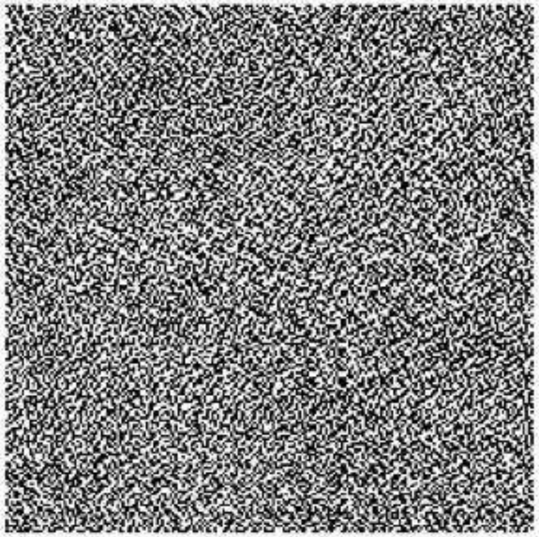
Inverse Filtering with Noise

Original Image

Degraded+noise Image



Restored Image



Pseudo-Inverse Filtering

- For an inverse filter, $\hat{F}(k,l) = \dfrac{G(k,l)}{H(k,l)}$.
- Here H(k,l) represents the spectrum of the PSF.
- The division of H(k,l) leads to large amplification at high frequencies and thus noise dominates over image

Pseudo-Inverse Filtering

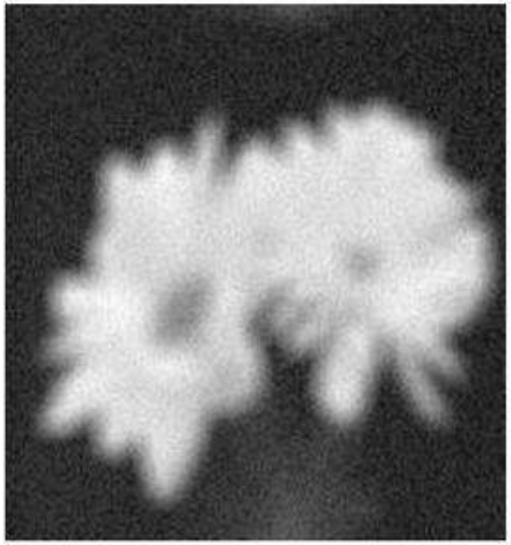
- To avoid this problem, a pseudo-inverse filter is defined as

$$\frac{1}{H} = \begin{cases} 1/H & \text{if } H > \varepsilon \\ \varepsilon & \text{if } H \le \varepsilon \end{cases}$$

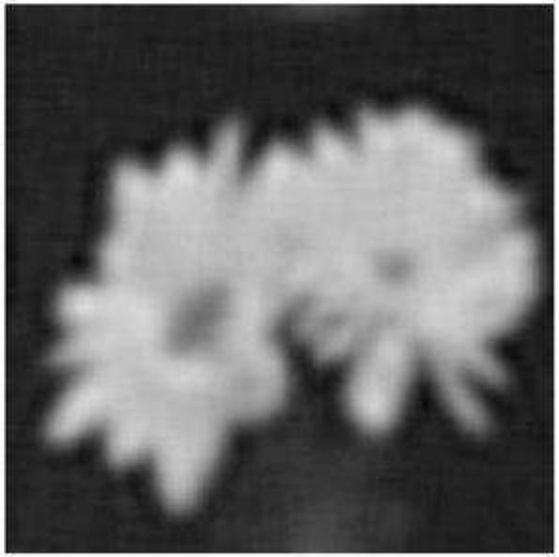
- The value of ε affects the restored image
- With no clear objective selection of ε, the restored images are generally noisy and not suitable for further analysis

Pseudo-Inverse Filtering

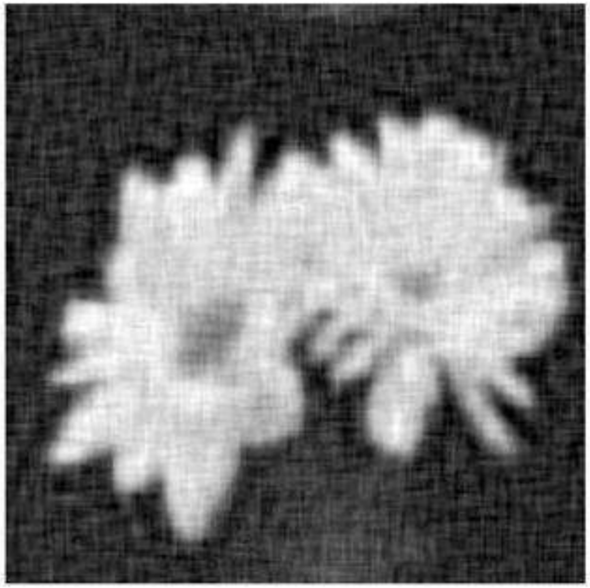

Original Image


Degraded+noise Image

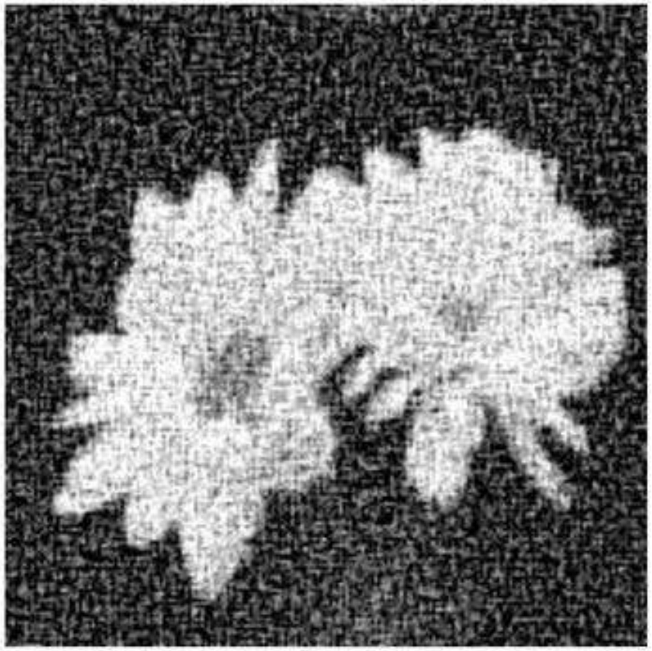
Pseudo-Inverse Filtering with ε = 0.2



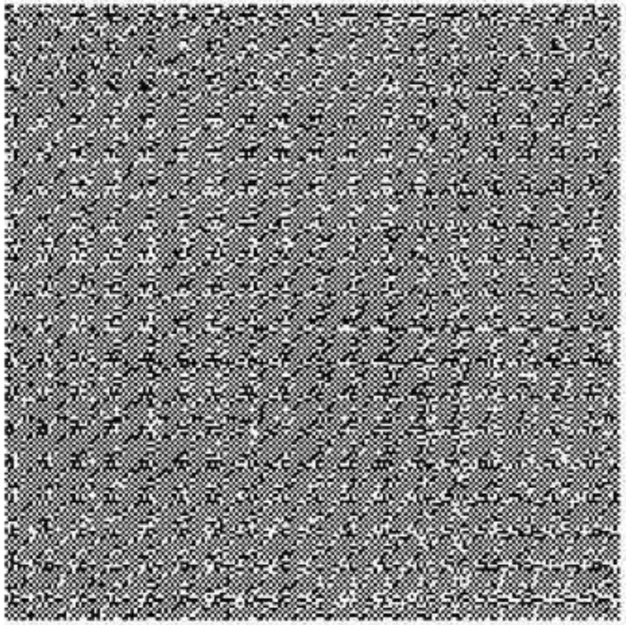
Pseudo-Inverse Filtering with ε = 0.02



Pseudo-Inverse Filtering with ε = 0



Pseudo-Inverse Filtering with ε = 0



SVD Approach for Pseudo-Inverse Filtering

- Advantage
  - Effective with noise amplification problem as we can interactively terminate the restoration
  - Computationally efficient if noise is space invariant
- Disadvantage
  - Presence of noise can lead to numerical instability

Thank You

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Topic : Wiener Filtering

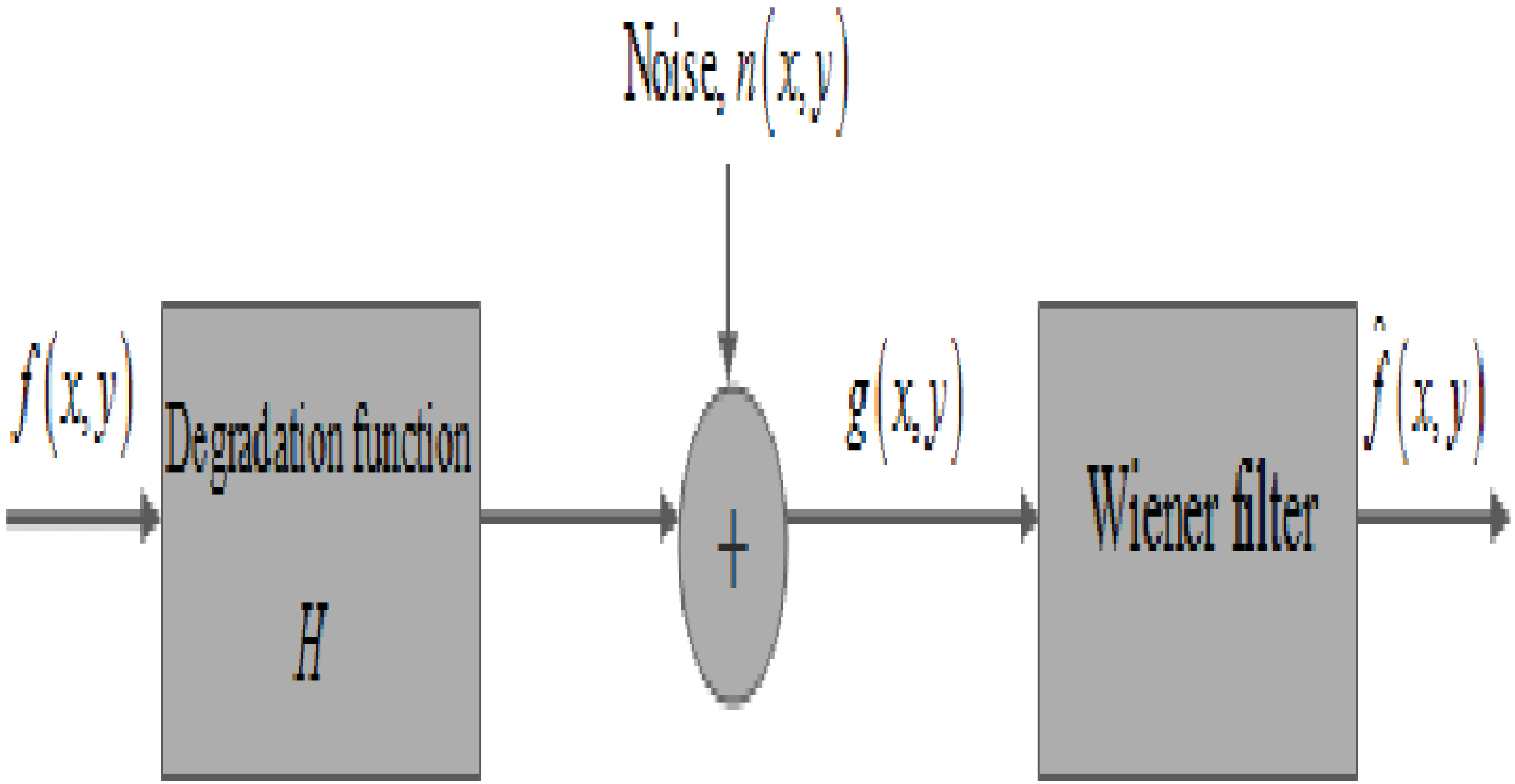
Wiener filtering was one of the first methods developed to reduce additive random noise in images. It works on the assumption that additive noise is a stationary random process, independent of pixel location; the algorithm minimizes the square error between the original and reconstructed images.

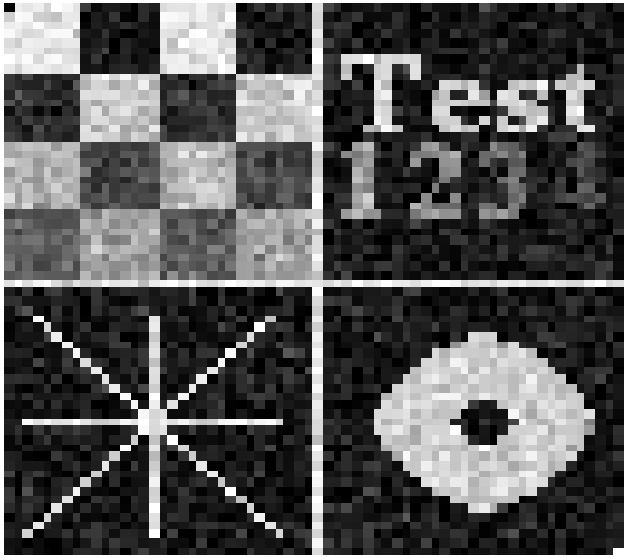
Wiener Filter

- The objective is to minimize the mean sqaure error
- It has the capability of handling both the degradation function and noise
- From the restoration model, the error between input image f(m,n) and the estimated image $(\hat{f}(m,n))$ is given by

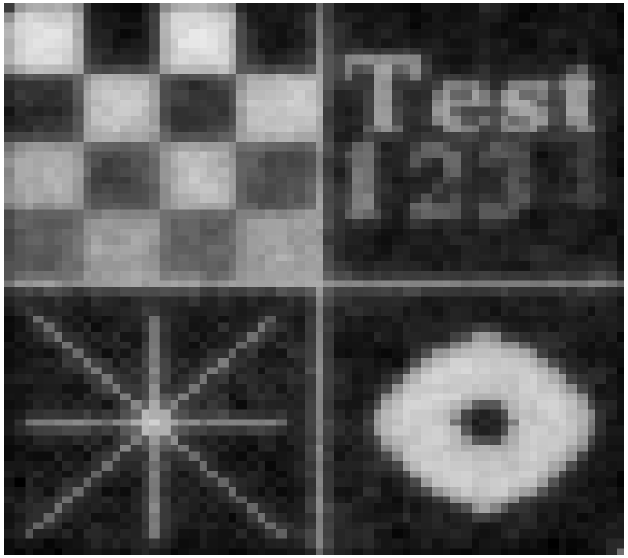
$$e(m,n) = f(m,n) - \hat{f}(m,n)$$

Wiener filters are comparatively slow to apply, since they require working in the frequency domain. To speed up filtering, one can take the inverse FFT of the Wiener filter G(u,v) to obtain an impulse response g(n,m). This impulse response can be truncated spatially to produce a convolution mask.


Noise, n(x,y)
f(x,y)
Degradation function
H
+
g(x,y)
Wiener filter
f̂(x,y)



Test image with AWGN
$\sigma_n^2 = 400$
but centered inside a
256x256 empty image



After Wiener filtering
MSE=121 (256x256 image)
MSE=1232 (portion shown)

Wiener Filter

- The square error is given by

$$[f(m,n)-\hat{f}(m,n)]^2$$

- The mean square error is given by

$$E\{[f(m,n)-\hat{f}(m,n)]^2\}$$

Wiener Filter

- The objective of the Wiener filter is top minimize $E\{[f(m,n) - \hat{f}(m,n)]^2\}$.
- Given a system we have

  y=h*x + v

  h-blur function

  x - original image

  y – observed image (degraded image)

  v – additive noise

Wiener Filter

- The goal is to obtain g such that

$$\hat{x} = g * y$$

$\hat{x}$ is the restored image that minimizes mean square error

- The deconvolution provides such a g(t)

Wiener Filter

- The filter is described in frequency domain as

$$G(f) = \frac{H*(f)S(f)}{|H(f)|^2 S(f) + N(f)}$$

- G and H are fourier transform of g and h
- S – mean power of spectral density of x
- N – mean power of spectral density of v
- * - complex conjugate

Wiener Filter

- Drawback – It requires prior knowledge of power spectral density of image which is unavailable in practice

Thank You