



UNIT 4 INTERPOLATION, NUMERICAL DIFFERENTIATION AND NUMERICAL  
INTEGRATION  
LAGRANGE'S INTERPOLATION

Interpolation, Numerical Differentiation and  
Numerical Integration.

Lagrange's Formula:

Let  $y_0, y_1, y_2, \dots, y_n$  be the corresponding values  
to  $x_0, x_1, x_2, \dots, x_n$ , then Lagrange's interpolation  
formula is,

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)\dots(x-x_n)}{(x_0-x_1)(x_0-x_2)\dots(x_0-x_n)} y_0 +$$

$$\frac{(x-x_0)(x-x_2)(x-x_3)\dots(x-x_n)}{(x_1-x_0)(x_1-x_2)\dots(x_1-x_n)} y_1 + \dots$$

$$+ \frac{(x-x_0)(x-x_1)\dots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\dots(x_n-x_{n-1})} y_n$$

1. Find the polynomial  $f(x)$  by using Lagrange's formula  
and hence find  $f(3)$  for

$x$	0	1	2	5
$f(x)$	2	3	12	47

$$x_0=0, x_1=1, x_2=2, x_3=5$$

$$y_0=2, y_1=3, y_2=12, y_3=47$$



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By using Lagrange's formula,

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

$$= \frac{(x-1)(x-2)(x-5)}{(0-1)(0-2)(0-5)} (2) + \frac{(x-0)(x-2)(x-5)}{(1-0)(1-2)(1-5)} (3)$$

$$+ \frac{(x-0)(x-1)(x-5)}{(2-0)(2-1)(2-5)} (12) + \frac{(x-0)(x-1)(x-2)}{(5-0)(5-1)(5-2)} (147)$$

$\Rightarrow x=3$

$$= \frac{(3-1)(3-2)(3-5)}{(-1)(-2)(-5)} (2) + \frac{(3-0)(3-2)(3-5)}{(1)(-1)(-4)} (3)$$

$$+ \frac{(3-0)(3-1)(3-5)}{(2)(1)(-3)} (12) + \frac{(3-0)(3-1)(3-2)}{(5)(4)(3)} (147)$$

$$= \frac{(2)(1)(-2)}{(-10)} (2) + \frac{(3)(1)(-2)}{4} (3) + \frac{(3)(2)(-2)}{-6} (12)$$

$$+ \frac{(3)(2)(1)}{60} (147)$$

$$= \frac{8}{10} - \frac{9}{2} + 24 + \frac{882}{60}$$

$$= 0.8 - 4.5 + 24 + 14.7$$

$$= 35$$





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2. Using Lagrange's Interpolation, calculate the profit in the year 2000 from the following data

Year	1997	1999	2001	2002
Profit in Lakhs Rs	43	65	159	248

$$x_0 = 1997, x_1 = 1999, x_2 = 2001, x_3 = 2002$$

$$y_0 = 43, y_1 = 65, y_2 = 159, y_3 = 248$$

By using Lagrange's formula,

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

$$= \frac{(x-1999)(x-2001)(x-2002)}{(1997-1999)(1997-2001)(1997-2002)} (43) + \frac{(x-1997)(x-2001)(x-2002)}{(1999-1997)(1999-2001)(1999-2002)} (65)$$

$$+ \frac{(x-1997)(x-1999)(x-2002)}{(2001-1997)(2001-1999)(2001-2002)} (159) + \frac{(x-1997)(x-1999)(x-2001)}{(2002-1997)(2002-1999)(2002-2001)} (248)$$

At  $x = 2000$

$$= \frac{(1)(-1)(-2)}{(-2)(-4)(-5)} (43) + \frac{(3)(-1)(-2)}{(2)(-2)(-3)} (65) + \frac{(3)(1)(-2)}{(4)(2)(-1)} (159) +$$

$$\frac{(3)(1)(-1)}{(5)(3)(1)} (248)$$



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$$= \frac{-86}{40} + \frac{390}{12} + \frac{954}{8} + \frac{(-744)}{15}$$

$$= 2.150 + 32.500 + 119.250 - 49.600$$

$$= 100.$$

2. Find the missing term in the following table using Lagrange's interpolation.

x	0	1	2	3	4
y	1	3	9	—	81

$x_0=0, x_1=1, x_2=2, x_3=3, x_4=4$

$y_0=1, y_1=3, y_2=9, y_3=81$

$$y=f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

$$= \frac{(3-1)(3-2)(3-4)}{(0-1)(0-2)(0-4)} (1) + \frac{(3-0)(3-2)(3-4)}{(1-0)(1-2)(1-4)} (3)$$

$$+ \frac{(3-0)(3-1)(3-4)}{(2-0)(2-1)(2-4)} (9) + \frac{(3-0)(3-1)(3-2)}{(4-0)(4-1)(4-2)} (81)$$





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$$\begin{aligned} &= \frac{(2)(1)(-1)}{(-1)(-2)(-4)} (1) + \frac{(3)(1)(-1)}{(1)(-1)(-3)} (3) + \frac{(3)(2)(1)}{(2)(1)(-2)} (9) \\ &\quad + \frac{(3)(2)(1)}{(4)(3)(2)} (81) \\ &= \frac{1}{4} - 3 + \frac{27}{2} + \frac{81}{4} \\ &= 0.25 - 3 + 13.5 + 20.25 \\ &= 31. \end{aligned}$$