UNIT 4 INTERPOLATION,NUMERICAL DIFFERENTIATION AND NUMERICAL INTEGRATION

LAGRANGE'S INTERPOLATION

Interpolation, Numerical Differentiation and Numerical Integration.

Lagrange's Formula:

Let $y_0, y_1, y_2 \dots y_n$ be the corresponding values to $x_0, x_1, x_2, \dots x_n$, then Lagrange's interpolation formula is,

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)\dots(x-x_n)}{(x_0-x_1)(x_0-x_2)\dots(x_0-x_n)} y_0 +$$

$$\frac{(x-x_0)(x-x_2)(x-x_3)\dots(x-x_n)}{(x_1-x_0)(x_1-x_2)\dots(x_1-x_n)} y_1 + \dots$$

$$+ \frac{(x-x_0)(x-x_1)\dots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\dots(x_n-x_{n-1})} y_n$$

1. Find the polynomial $f(x)$ by using Lagrange's formula and hence find $f(3)$ for

x	0	1	2	5
$f(x)$	2	3	12	47

$x_0 = 0, x_1 = 1, x_2 = 2, x_3 = 5$

$y_0 = 2, y_1 = 3, y_2 = 12, y_3 = 47$

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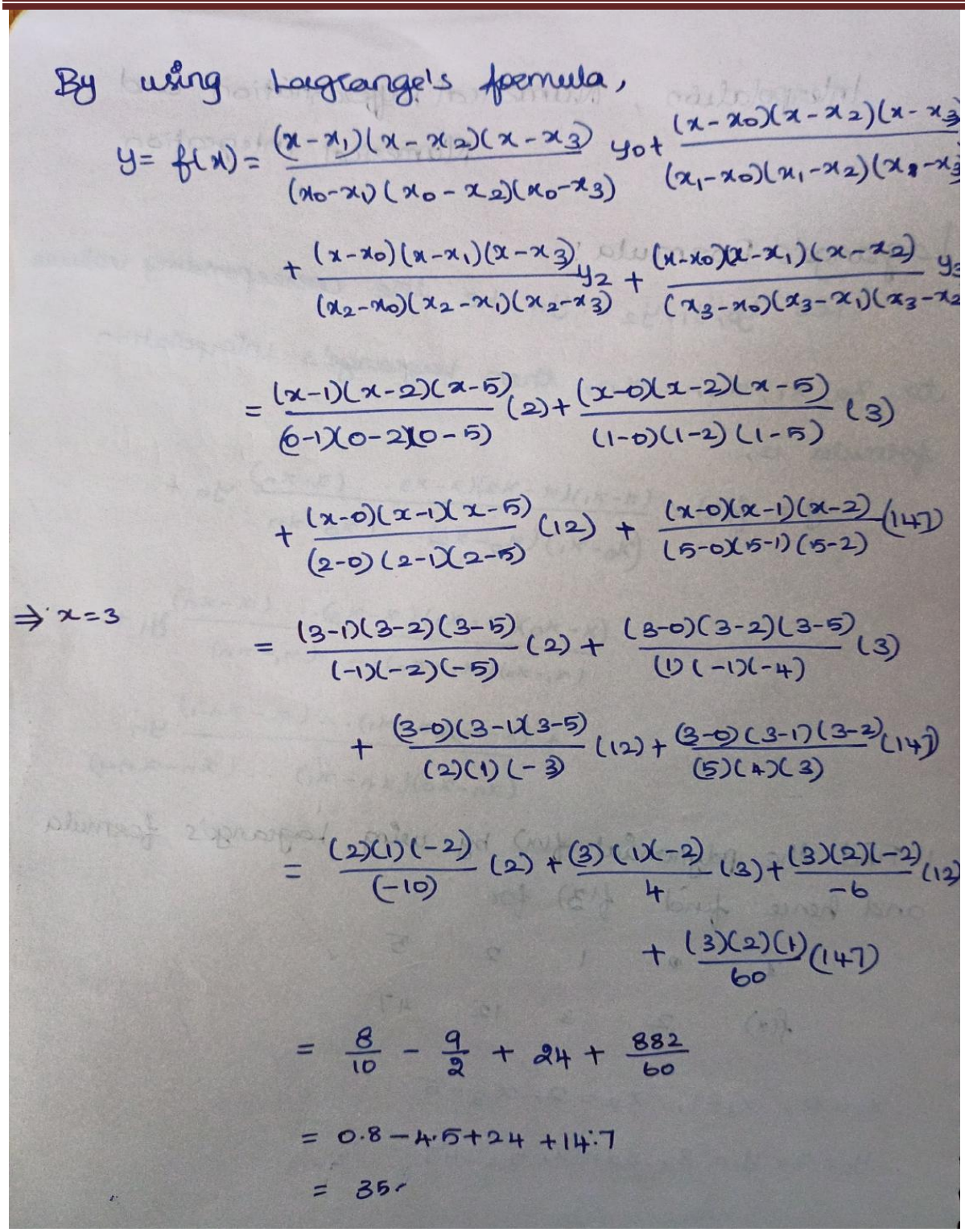

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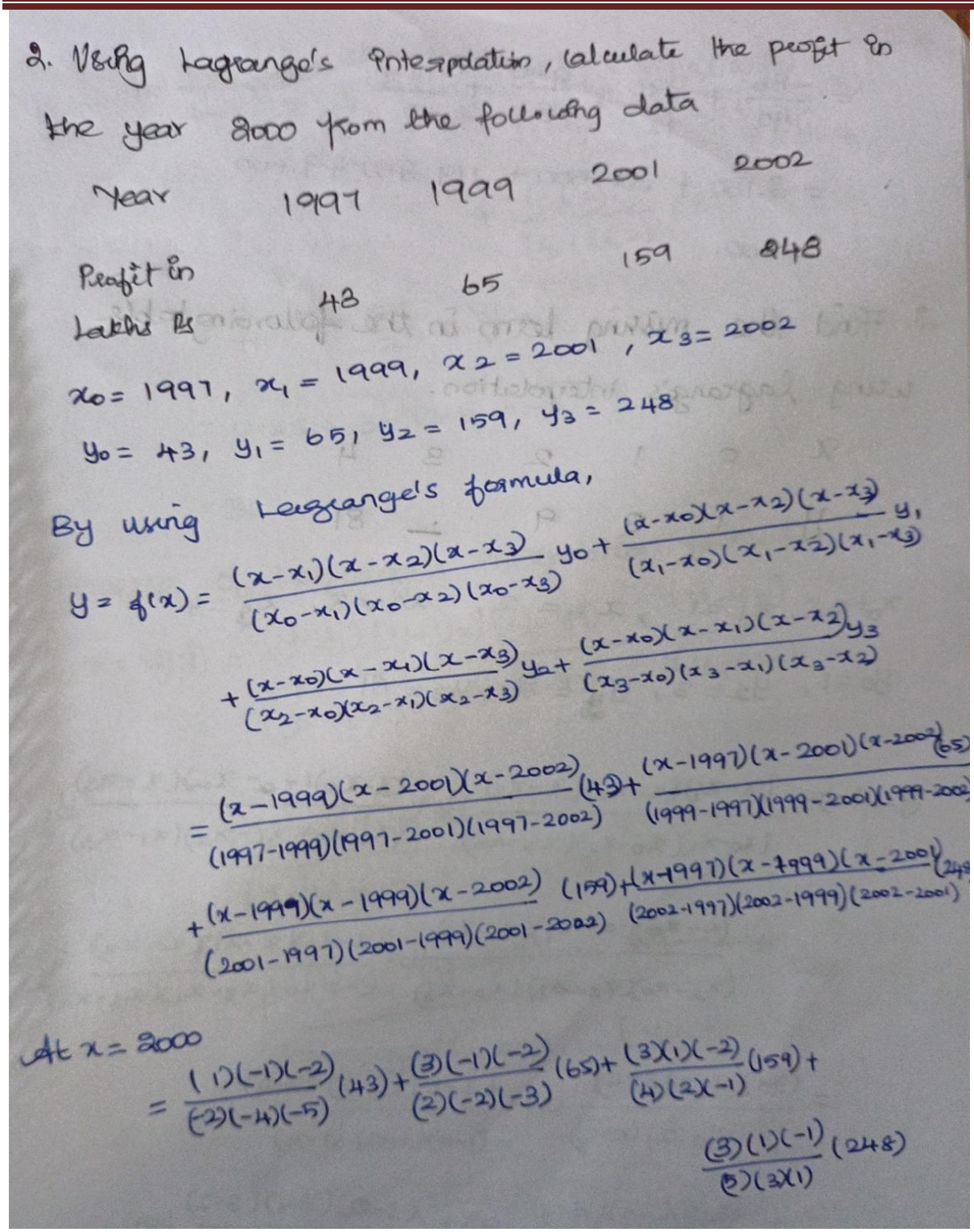

2. Using Lagrange's Interpolation, calculate the profit in the year 2000 from the following data

Year	1997	1999	2001	2002
Profit in Lakhs Rs	43	65	159	248

$x_0 = 1997,\ x_1 = 1999,\ x_2 = 2001,\ x_3 = 2002$

$y_0 = 43,\ y_1 = 65,\ y_2 = 159,\ y_3 = 248$

By using Lagrange's formula,

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}y_1$$

$$+\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}y_3$$

$$= \frac{(x-1999)(x-2001)(x-2002)}{(1997-1999)(1997-2001)(1997-2002)}(43) + \frac{(x-1997)(x-2001)(x-2002)}{(1999-1997)(1999-2001)(1999-2002)}(65)$$

$$+\frac{(x-1999)(x-1999)(x-2002)}{(2001-1997)(2001-1999)(2001-2002)}(159) + \frac{(x-1997)(x-1999)(x-2001)}{(2002-1997)(2002-1999)(2002-2001)}(248)$$

At $x = 2000$

$$= \frac{(1)(-1)(-2)}{(-2)(-4)(-5)}(43) + \frac{(3)(-1)(-2)}{(2)(-2)(-3)}(65) + \frac{(3)(1)(-2)}{(4)(2)(-1)}(159) +$$

$$\frac{(3)(1)(-1)}{(5)(3)(1)}(248)$$

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$$= \frac{-86}{40} + \frac{390}{12} + \frac{954}{8} + \frac{(-744)}{15}$$

$$= 2.150 + 32.500 + 119.850 - 49.600$$

$$= 100.$$

2. Find the missing term in the following table using lagrange's interpolation.

x	0	1	2	3	4
y	1	3	9	—	81

$x_0 = 0$, $x_1 = 1$, $x_2 = 2$, $x_3 = 3$, $x_4 = 4$

$y_0 = 1$, $y_1 = 3$, $y_2 = 9$ $y_3 = 81$

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

$$= \frac{(3-1)(3-2)(3-4)}{(0-1)(0-2)(0-4)}(1) + \frac{(3-0)(3-2)(3-4)}{(1-0)(1-2)(1-4)}(3)$$

$$+ \frac{(3-0)(3-1)(3-4)}{(2-0)(2-1)(2-4)}(9) + \frac{(3-0)(3-1)(3-2)}{(4-0)(4-1)(4-2)}(81)$$

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